USTARS

underrepresented students in topology and algebra research symposium

Amherst College
Amherst, Massachusetts
March 31- April 2, 2017

Agenda

Friday, March 31, 2017

6:00am -10:00pm **Arrival to USTARS March 31, 2017**
Location: Howard Johnson Hotel

6:00pm-8:00pm **Conversations with a Mathematician**
Dr. Julianna Tymoczko, Smith College
Location: Seeley Mudd 206

Saturday, April 1, 2017

8:00am -12:00pm **Registration**
Location: Seeley Mudd 208

9:30am -9:45am **Welcome and Opening Remarks**
Dr. Gabriel Sosa, Amherst College
Location: Stim Auditorium (Mead Art Museum)

10:00am - 10:30am **Session I**
Khovanov Homology for Almost Alternating Knots
Gabriel Montoya-Vega, University of Puerto Rico-Mayaguez
Location: Seeley Mudd 204

Peri-Catalan Numbers: A Tale of Combinatorics and Nonassociativity
Stefanie Wang, Iowa State University
Location: Seeley Mudd 206

Studying Discretization Error with Lie Algebras and Lie Groups
Wiliam Ty Frazier, East Tennessee State University
Location: Seeley Mudd 207

10:40 -11:10am **Session II**
Infinite games and the quasi-uniform box product
Hope Sabao, North-West University
Location: Seeley Mudd 204

Centralizer Algebras of Unipotent Upper Triangular Matrices
Megan Ly, University of Colorado Boulder
Location: Seeley Mudd 206

Effective Residual Finiteness of Lattices in Nilpotent Lie Groups
Mark Pengitore, Purdue University
Location: Seeley Mudd 207

Saturday, April 1, 2017

11:20-11:50am	**Session III** *First Bounds on the Dilatation of Pure Surface Braids* Marissa Loving, University of Illinois at Urbana-Champaign Location: Seeley Mudd 204 *Bivariate Order Polynomials* Sandra Zuniga-Ruiz, San Francisco State University Location: Seeley Mudd 206 *Mathematical Structures in Musical Spaces* Yvonne Chazal, North Carolina State University Location: Seeley Mudd 207
12:00-2:00pm	**Lunch and Mentoring Panel** Location: Lewis-Sebring Commons **Panelists:** Nick Camacho, University of Iowa Cory Colbert, Williams College/ University of Texas, Austin Dr. Van Nguyen, Northeastern University Dr. Chris Oneill, University of California, Davis Dr. Jeremiah Reinkoester, State Farm Insurance Company **Moderator:** Dr. Syvillia Averett, College of Coastal Georgia
2:00-3:30pm	**Invited Faculty Speaker** *Math in the Age of Trump* Dr. Chelsea Walton, Temple University Location: Stim Auditorium (Mead Art Museum)
3:45-5:00pm	**Poster Session and Informal Networking** Location: TBA
5:10-5:30pm	**Group Photo** Location: Seeley Mudd 204
5:40-6:40pm	**Distinguished Graduate Speaker** *Uniform Asymptotic Growth on Symbolic Powers of Ideals* Robert Walker, University of Michigan Location: Stim Auditorium (Mead Art Museum)
7:00-8:30pm	**Symposium Banquet** Location: Lewis-Sebring Commons

Sunday, April 2, 2017

9:30-10:00am	**Session IV** *Geometry of Equilateral Knot Space* Kate Hake, University of California, Santa Barbara Location: Seeley Mudd 204 *The Equivariant Ehrhart Theory of the Permutahedron* Andrés Vindas Meléndez, San Francisco State University Location: Seeley Mudd 206 *The Boson-Fermion Correspondence* Nicolle Sandoval Gonzalez, University of Southern California Location: Seeley Mudd 207
10:10am-10:40am	**Session V** *The Glasner-Pestov problem and examples of extremely amenable groups* Javier Ronquillo Rivera, Ohio University Location: Seeley Mudd 204 *Quandles of Virtual Knots* Sherilyn Tamagawa, University of California, Santa Barbara Location: Seeley Mudd 206 *Stabilizing Spectral Functors of Exact Categories* Juan Villeta-Garcia, University of Illinois at Urbana-Champaign Location: Seeley Mudd 207
10:50 - 11:50am	**Distinguished Graduate Speaker** *Symplectic mapping class group relations generalizing the chain relation* Bahar Acu, University of Southern California/ University of California Los Angeles Location: Stim Auditorium (Mead Art Museum)
12:00-12:45pm	**Lunch and Symposium Closing** Location: Lewis-Sebring Commons

USTARS 2017 Invited Faculty

USTARS 2017 Invited Faculty Speaker

Dr. Chelsea Walton, Temple University

Dr. Chelsea Walton was born and raised in Detroit, Michigan, attended Detroit Public Schools, then enrolled at Michigan State University, graduating in 2005 with a B.S. with High Honors in Mathematics. While at MSU, Dr. Walton participated in an alternative spring break program (volunteer work) in Puebla, Mexico and a Race Relations summer study abroad program in South Africa. These experiences fostered her interests in race relations and diversity, especially in increasing the representation of underrepresented groups in the mathematical sciences.

After completing her graduate work at the University of Michigan, Dr. Walton was awarded an NSF postdoc at the University of Washington in Seattle in 2011-2012; her postdoc mentor was James Zhang. In Spring 2013 she participated in the program on Noncommutative Algebraic Geometry and Representation Theory at MSRI under the mentorship of Sarah Witherspoon. Finally, her postdoc experience ended at MIT, where, under the guidance of Pavel Etingof, she was Moore Instructor in the Math Department.

Dr. Walton is currently Selma Lee Bloch Assistant Professor in the Mathematics Department at Temple University in Philadelphia, Pennsylvania.

USTARS 2017 "Conversations With a Mathematician" Invited Lecturer

Dr. Julianna Tymoczko, Smith College

Dr. Julianna Tymoczko is an Associate Professor of Mathematics at Smith College in Northampton, Massachusetts. Dr. Tymoczko began her undergraduate studies at Smith College before earning her degree from Harvard-Radcliffe. In 2003 she earned her doctorate from Princeton University. Her mathematical journey has taken her to various states in the midwest including Michigan and Iowa. In 2011, she returned as a faculty member to the mathematics department at Smith College.

USTARS 2017 Distinguished Graduate Students

USTARS 2017 Distinguished Graduate Student in Algebra

Robert Walker, University of Michigan

I originally hail from Urbana, Illinois (like the robot actor Calculon in Futurama). My lifelong dream since childhood is to work on a graphic novel. I went to undergrad in my hometown and majored in both math and philosophy. I am a Caesarian-born Libra (so I could slay Macbeth), and I share a birthday with Neil Degrasse Tyson (30 year difference) and Fields medalist Cedric Villani (15 year difference). My middle name is Marshawn, like the football player, and I often go by "Mars" with my students. I usually tell non-math people ("math muggles") that I am a comedian of abstract logic (an "Ar-comedian"). I am in my final year as an NSF Grad Fellow, advised by Karen Ellen Smith in the UMich-Ann Arbor math department; Karen gave the 2016 AWM Noether Lecture at the Seattle JMM. I will graduate in 2018 and hope to end up in a liberal bastion on the West Coast somewhere. My favorite number is 3125, but I'll try to use the perfect number 28 in my talk.

USTARS 2017 Distinguished Graduate Student in Topology

Bahar Acu, University of Southern California/ University of California Los Angeles

I was born and raised in the Kurdish (an ethnic minority in Middle East largely populated in Turkey, Iraq, Iran, and Syria) part of Turkey as the youngest child of a family with seven children. My siblings and I are the first generation to go to college on both my mothers and fathers sides. Since I was the last child, I was fortunate to have had the opportunity to get inspired by my siblings and their achievements. As an underrepresented individual, both ethnically and professionally in my field, I have always believed that our limits are more flexible and our small or big achievements are more inspirational than we think they are.

I am currently a visiting graduate researcher at University of California, Los Angeles and a PhD candidate at University of Southern California who recently defended her dissertation! Yay! I received my BS and MS degrees from Middle East Technical University in Turkey and MS degree from University of Southern California. I will start working as a Ralph Boas Assistant Professor of Mathematics at Northwestern University beginning Fall 2017. My research spans geometry and topology of higher dimensional contact and symplectic manifolds and low dimensional topology. I am currently the president of the USC Student Chapter of AWM and an executive board member of the Charlottes Web: women in mathematics group at USC.

Presentation Abstracts

Title: Symplectic mapping class group relations generalizing the chain relation
Presenter: Bahar Acu*
Affiliation: Contact geometry, also known as the 21st century geometry, is the study of a geometric structure on an odd dimensional smooth topological manifold given by a certain type of hyperplane distribution. It has been seen to underlie many physical phenomena and is motivated by mathematical formalism of classical mechanics. In this talk, we will examine symplectomorphisms" of higher dimensional "symplectic manifolds (even dimensional counterpart of contact manifolds) by using "fibered Dehn twists. Despite the fact that many fibered Dehn twists cannot be expressed as a product of "Dehn twists", we are still able to show a fibered Dehn twist is symplectically isotopic to a product of Dehn twists along Lagrangian spheres. Moreover, we show that this identification yields a generalization of the classical chain relation on surfaces. This is joint work with Russell Avdek. No background in contact/symplectic topology and geometry will be required as most of the concepts will be defined during the talk.

Title: Mathematical Structures in Musical Spaces
Presenter: Yvonne Chazal
Affiliation: North Caronlina State University
Abstract: It's been known for centuries that there are many connections between mathematics and music. D. Tymoczko described and utilized the representation of n-note chords by n-dimensional orbifolds, opening up many possibilities for musical analysis. A. Crans et al. formulate dihedral group actions from musical concepts whose commutativity is present in examples that span centuries. In this talk, we will focus on analyzing music via topological structures arising in Tymoczko's model and the group actions described by Crans, as well as the interactions between the two models.

Title: Studying Discretization Error with Lie Algebras and Lie Groups
Presenter: William Ty Frazier
Affiliation: East Tennessee State University
Abstract: Molecular Dynamics (MD) is the numerical simulation of a large system of interacting molecules, and one of the key components of an MD simulation is the numerical estimation of the solutions to a system of nonlinear differential equations. Such systems are very sensitive to discretization and round off error, and correspondingly, standard techniques such as Runge-Kutta methods can lead to poor results. However, MD systems are conservative, which means that we can use Hamiltonian mechanics and symplectic transformations (also known as canonical transformations) in analyzing and approximating solutions. This is standard in MD applications, leading to numerical techniques known as symplectic integrators, and often, these techniques are developed for well-understood Hamiltonian systems such as Hills lunar equation. In this presentation, we explore how well symplectic techniques developed for well-understood systems (specifically, Hills Lunar equation) address discretization errors in MD systems which are fail for one or more reasons.

Title: Geometry of Equilateral Knot Space
Presenter: Kate Hake
Affiliation: University of California, Santa Barbara
Abstract: A n-sided polygon in $\mathbb{R}^3$ can be described as a point in $\mathbb{R}^{3n}$ by listing in order the coordinates of it vertices. In this way, the space of n-sided polygons embedded in $\mathbb{R}^3$ is a manifold in which points correspond to piecewise linear knots and paths correspond to isotopies which preserve the geometric structure of these knots. Restricting to polygons of unit edge length gives a submanifold consisting of equilateral knots. We will discuss some aspects of the topology of the space of equilateral hexagons as well as its symplectic structure.

Title: First Bounds on the Dilatation of Pure Surface Braids
Presenter: Marissa Loving
Affiliation: University of Illinois at Urbana-Champaign

Abstract: We provide a brief introduction to the notion of pure surface braids and dilatation of mapping classes. We then provide motivation for our initial bounds on the dilatation of pure surface braids and examples in the case of the point pushing subgroup.

Title: Centralizer Algebras of Unipotent Upper Triangular Matrices
Presenter: Megan Ly
Affiliation: University of Colorado Boulder
Abstract: Classical Schur-Weyl duality relates the irreducible characters of the symmetric group S_n to the irreducible characters of the general linear group $GL_n(\mathbb{C})$ via their commuting actions on tensor space. We investigate the analog of this result for the group of unipotent upper triangular matrices $UT_n(\mathbb{F}_q)$. In this case the character theory of $UT_n(\mathbb{F}_q)$ is unattainable, so we must employ supercharacter theory, creating a striking variation.

Title: Khovanov Homology for Almost Alternating Knots
Presenter: Gabriel Montoya-Vega
Affiliation: University of Puerto Rico- Mayaguez
Abstract: Although knot theory has been around since the end of the nineteenth century, it gained steam and produced some of its most exciting results within the last thirty years. In this way, being the classification the main aim of this theory, the large quantity of almost alternating knots gives rise to an important category. Following a result in Adams (1992) esteemed paper on almost alternating knots, we assume the task of finding which specific knots differ from the others satisfying an equation and the reasons why these occurs. In order to detect the knots, we established a result previously given for the span of the bracket polynomial for almost alternating knots, in terms of the Jones polynomial. The Khovanov complex, conducive to the homology, of a given knot K is generated by considering a planar projection of the knot with 2^n states, each of which consists of a collection of simple closed curves on the plane. Using the fact that Khovanov homology is a strong invariant of knots, we use Mathematica as a tool to find out some aspects that made each knot unique. Finally, we show the relevant knots and the facts that, in contrast with the others, make them not hold the equation, giving special attention to the polynomial generated by the Khovanov complex.

Title: Effective Residual Finiteness of Lattices in Nilpotent Lie Groups
Presenter: Mark Pengitore
Affiliation: Purdue University
Abstract: We compute the complexity of finding a surjective homomorphism to a finite group from a cocompact lattice in a connected, simply connected nilpotent Lie group such that a fixed non-trivial element is not in the kernel. We also demonstrate that the complexity only depends on the ambient nilpotent Lie group.

Title: The Glasner-Pestov problem and examples of extremely amenable groups
Presenter: Javier Ronquillo Rivera
Affiliation: Ohio University
Abstract: When a class of abelian topological groups has 'enough' non-trivial characters we can study this class of topological groups through a duality theory. A classical example of this is the theory of Potryagin duality for locally compact abelian groups. Nonetheless for some classes of abelian topological groups the question about having 'enough' non-trivial characters remains open. One of these open problems is due to Glasner and Pestov and states the following:

Question 1 (Glasner-Pestov problem)**.** *Let X be compact, $G \subset \mathrm{Homeo}(X)$ an abelian group, such that X has no G-fixed points. Does G admit non-trivial characters?*

One approach to this problem is studying extremely amenable groups. A topological group G is said to be **extremely amenable** if G has a $G-fixed$ point in every compact set X it acts upon. It is easy to see that if G is abelian and extremely amenable then G admits no non-trivial characters.

The converse of this proposition remains open and indeed is a reformulation of the Glasner-Pestov problem:

Question 2. *Are all abelian topological groups G that do not admit a non-trivial character extremely amenable?*

In this talk we will look at some of the results mentioned above and some examples of extremely amenable groups.

Title: Infinite games and the quasi-uniform box product
Presenter: Hope Sabao
Affiliation: North-West University
Abstract: We generalise the two-player infinite game played in a uniform space to the framework of quasi-uniform spaces. We then use this game to show that the quasi-uniform box-product of countably many copies of a Fort-space is collectionwise normal countably paracompact and collectiowise Hausdorff.

Title: The Boson-Fermion Correspondence
Presenter: Nicolle Sandoval Gonzalez
Affiliation: University of Southern California
Abstract: The boson-fermion correspondence lies at the heart of quantum mechanics and combinatorial representation theory. In essence, it encodes the fact that the Heisenberg algebra can be constructed from the Clifford algebra and vice versa. Given that the action of the Clifford algebra on fermionic Fock space mimics that of annihilation and creation operators in the Dirac sea of electrons and the seemingly disjoint nature of bosons and fermions, this connection is naturally of great interest to quantum physicists. Mathematically speaking, it reveals how the action of the Clifford algebra on fermionic Fock space can be recovered from the action of the Weyl algebra on bosonic Fock space. Given the combinatiorial nature of both spaces and their well known representations of the Weyl and Clifford algebras, this correspondence gives immediate insight into the rich interplay between representation theory, combinatorics, and quantum physics. In this talk we will introduce these topics from an algebraic point of view and mention some attempts of their categorification.

Title: Quandles of Virtual Knots
Presenter: Sherilyn Tamagawa
Affiliation: University of California Santa Barbara
Abstract: We will explore algebraic invariants on a special class of knots, known as virtual knots. Our main focus will be on quandles, an algebraic structure which serves as a perfect knot invariant, but is usually impossible to compute. We construct new invariants by taking quotients of quandles, and introduce a variation on the Alexander polynomial.

Title: Stabilizing Spectral Functors of Exact Categories
Presenter: Juan Villeta-Garcia
Affiliation: University of Illinois at Urbana-Champaign
Abstract: Algebraic K-Theory is often thought of as the universal additive invariant of rings (or more generally, exact categories). Often, however, functors on exact categories dont satisfy additivity. We will describe a procedure (due to McCarthy) that constructs a functors universal additive approximation, and apply it to different different local coefficient systems, recovering known invariants of rings (K-Theory, THH, etc.). We will talk about what happens when we push these constructions to the world of spectra, and tie in work of Lindenstrauss and McCarthy on the Taylor tower of Algebraic K-Theory.

Title: The Equivariant Ehrhart Theory of the Permutahedron
Presenter: Andrés Vindas Meléndez
Affiliation: San Francisco State University
Abstract: Ehrhart theory is a well-established field that studies lattice points in dilations of polytopes. Polytopes are geometric objects with flat sides that exist in all dimensions. Much of the structure connecting the

volume of a dilated polytope to the number of lattice points it contains is encoded in its Ehrhart polynomial. In 2010, Alan Stapledon described an equivariant analogue to Ehrhart theory as an extension of the theory with group actions. My joint work with Anna Schindler, attempts to describe the equivariant Ehrhart theory of the permutahedron, which is the convex hull of all points formed by permuting the coordinates of the vector $(1, 2, ..., d)$. In this talk, I will present our results that include information on the dimension of the fixed polytopes, combinatorial equivalence, and the relationship to zonotopes.

Title: Uniform Asymptotic Growth on Symbolic Powers of Ideals
Presenter: Robert Walker*
Affiliation: University of Michigan
Abstract: Algebraic geometry (AG) is a major generalization of linear algebra which is fairly influential in mathematics. Since the 1980's with the development of computer algebra systems like Mathematica, AG has been leveraged in areas of STEM as diverse as statistics, robotic kinematics, computer science/geometric modeling, and mirror symmetry. Part one of my talk will be a brief introduction to AG, to two notions of taking powers of ideals (regular vs symbolic) in Noetherian commutative rings, and to the ideal containment problem that I study in my thesis. Part two of my talk will focus on stating the main results of my thesis in a user-ready form, giving a "comical" example or two of how to use them. At the risk of sounding like Paul Rudd in *Ant-Man*, I hope this talk with be awesome.

Title: Math in the Age of Trump
Presenter: Dr. Chelsea Walton**
Affiliation: Temple University
Abstract:

Title: Peri-Catalan Numbers: A Tale of Combinatorics and Nonassociativity
Presenter: Stefanie Wang
Affiliation: Iowa State University
Abstract: A quasigroup is an algebra equipped with nonassociative binary operations of multiplication, left division, and right division. Rooted binary trees with n leaves represent the number of ways to bracket a word with n arguments involving a single nonassociative binary operation. While Catalan numbers count rooted binary trees with n leaves, we are interested in the peri-Catalan numbers that count the number of quasigroup words with n arguments. We will provide an introduction to quasigroups and the motivation behind the study of peri-Catalan numbers.

Title: Bivariate Order Polynomials
Presenter: Sandra Zuniga-Ruiz
Affiliation: San Francisco State University
Abstract: In the early 1900s, in an attempt to prove the 4-coloring theorem Birkhoff discovered the number of ways to color a graph, known as the chromatic polynomial. In 1970, Richard Stanley found a way to decompose the chromatic polynomial using order polynomials along with many other interesting results. In 2003, Klaus Dohmen, Andre Pnitz, and Peter Tittman introduced a two variable generalization of the chromatic polynomial. We define bivariate order polynomials of bicolored posets. Furthermore, we provide a way to compute the bivariate order polynomial of any given bicolored poset. We also present the decomposition the bivariate chromatic polynomial using bivariate order polynomials.

** Invited Faculty Speaker
* Distinguished Graduate Student

Poster Abstracts

Title: An example of homological mirror symmetry
Presenter: Catherine Cannizzo
Affiliation: University of California Berkeley
Abstract: This thesis project aims to prove the homological mirror symmetry conjecture (HMS) for the genus 2 curve inside an abelian surface. Mirror symmetry states that certain manifolds come in pairs, such that the complex geometry on one corresponds to the symplectic geometry on the other. The homological version, due to M. Kontsevich, says that the bounded derived category of coherent sheaves on the complex manifold is equivalent to the Fukaya category on the symplectic manifold. The bounded derived category of coherent sheaves reduces to looking at line bundles in our case, and the Fukaya category is built from Lagrangians as objects and their intersection points as morphisms. The techniques used in this project are similar to those in the ongoing work of M. Abouzaid and D. Auroux for HMS for hypersurfaces in toric varieties, the difference here being that we are working with a hypersurface of an abelian variety i.e. of a toric variety modulo a group action. This is currently work in progress.

Title: Lengths of Simple Closed Curves on the Punctured Torus
Presenter: Sabrina Enriquez
Affiliation: University of Southern California
Abstract: I am studying the behavior of the lengths of simple closed geodesics on the surface of a punctured torus endowed with a hyperbolic metric. In particular, my goal is to identify the maximum and minimum values taken by the Mirzakhani function, over the space of all hyperbolic metrics on the punctured torus. The Mirzakhani function, introduced by Maryam Mirzakhani, controls the growth rate of lengths of simple closed curves on hyperbolic surfaces. Through a heuristic method, I have found that the minimum of the Mirzakhani function seems to be attained at the (unique) hyperbolic metric that has a symmetry of order 3. The function is also unbounded.

Title: Constructing Symplectic Varieties Using Geometric Invariant Theory
Presenter: Nicolette Jimenez
Affiliation: United States Military Academy at Westpoint
Abstract: I will provide strategies on how one constructs new projective varieties using classical ones. In the case when $n = 2$, I will show that the Hamiltonian reduction of $\mathfrak{b} \times \mathbb{C}^n$ is a complete intersection by using a well-known technique in symplectic geometry. I will construct four projective varieties using geometric invariant theory (GIT). I will also construct an affine variety via the affine quotient using the four irreducible components. Finally, I will discuss how one should construct a hyperkahler metric on these types of varieties.

Title: A closed formula for weight q-multiplicities for the representations of the Lie algebra $\mathfrak{sp}_4(\mathbb{C})$
Presenter: Edward Luber
Affiliation: Williams College
Abstract: In this poster we present a new closed formula for the values of the q-analog of Kostant's partition function for the Lie algebra $\mathfrak{sp}_4(\mathbb{C})$. We use this result to give a simple formula for the q-multiplicity of a weight in the representations of the Lie algebra $\mathfrak{sp}_4(\mathbb{C})$. This generalizes the 2012 work of Refaghat and Shahryari which presented a closed formula for weight multiplicities in representations of the Lie algebra $\mathfrak{sp}_4(\mathbb{C})$, and the 2015 results of Fernández-Núñez, García-Fuertes, and Perelomovto which presented a generating function for these multiplicities.

Title: Fundamental weight lattice patterns arising from the support of Kostant's weight multiplicity formula on $\mathfrak{sl}_3(\mathbb{C})$
Presenters: Haley Lescinsky and Grace Mabie
Affiliation: Williams College
Abstract: This work is motivated by the representation theory of the Lie algebra $\mathfrak{sl}_3(\mathbb{C})$ and the computation of the multiplicity of a weight μ in a finite-dimensional irreducible representation indexed by the dominant integral weight λ. This multiplicity, defined as the dimension of a particular vector space, can be computed via Kostant's weight multiplicity formula (KWMF). This formula consists of an alternating sum over a finite group called the Weyl group and involves a partition function. In practice, it has been observed that many terms of KWMF contribute a value of zero (i.e. trivially). Hence, our work involves determining the elements of the Weyl group that contribute nontrivially to KWMF and the the set of such elements is the called the Weyl alternation set and is denoted by $\mathcal{A}(\mu, \lambda)$. Taking a geometric approach we provide concrete descriptions of the sets $\mathcal{A}(\lambda, \mu)$ for all pairs of integral weights λ and μ of $\mathfrak{sl}_3(\mathbb{C})$. We also provide a visualization of these results via *Weyl alternation set diagrams*, in which we fix a weight μ and color code integer lattice points on the fundamental weight lattice of $\mathfrak{sl}_3(\mathbb{C})$ (corresponding to the integral weight λ) depending on the distinct Weyl alternation sets $\mathcal{A}(\lambda, \mu)$. We show that the diagrams associated to our main result present new and surprising symmetry patterns.

Title: Involutions and Orientation-preserving Symmetries in the Hyperoctahedral Group
Presenters: Jingtai Liu and Rita Post
Affiliation: University of Wisconsin Eau Claire
Abstract: We study a variety of permutation statistics in the hyperoctahedral group $\mathbb{G}(2, 1, n)$. We study a variety of involutions and orientation-preserving symmetries in the hyperoctahedral group $\mathbb{G}(2, 1, n)$. We examine the involutory elements and their corresponding conjugacy classes. We explore some combinatorical aspects of the analogue of the alternating subgroup $A_n \subseteq \mathfrak{S}_n$ in $\mathbb{G}(2, 1, n)$—namely, the orientation-preserving symmetries, which we denote $\mathbb{A}(2, 1, n)$. We present formulas for calculating the sizes of each involutory conjugacy class and for calculating the number of involutory conjugacy classes in $\mathbb{G}(2, 1, n)$ for all n. We investigate the intersection of the elements of $\mathbb{A}(2, 1, n)$ with the involutory elements for a given n in $\mathbb{G}(2, 1, n)$ to determine the number of orientation-preserving symmetries of order 2 for a given n.

Title: Geometry of Discriminants over Finite Fields: The geometric relationship of polynomials to the discriminant in Finite Fields
Presenter: Marcus Morales
Affiliation: University of New Mexico
Abstract: In the Gabriel Katz paper called How tangents solve algebraic equations, he concludes that the problem of solving a polynomial equation P(z) = 0 turns out to be equivalent to finding hyperplanes tangent to the polynomials discriminant and through a given point P in the space of coefficients. It follows that this conclusion holds for any field. Throughout this paper we will explore this relationship over a finite field. Although the idea of "tangency" in finite fields is not necessarily define as in the real case, the formal derivative allows us to preserve the behavior of a derivative without limits. Therefore the notion of a tangent line and the process to deriving it remains the same. Hence, the relationship between the problem of finding roots and the problem of finding tangent lines to the discriminant curve will still hold in any field, and in particular our case, the finite fields F_q, where $q = p^k$, p is a prime idea and k is the degree of. To depict these results we have created affine planes over $\mathbb{F}_q$ and considered all the monic quadratics in the field to determine: is it on the discriminant, does it have two unique solutions, or no solution? We have also visualized this data as a fractal to more naturally reflect the structure of $\mathbb{F}_q$ and hence more efficiently visualize the symmetry in the results. These results attempt to connect/relate Katz' geometric results that connect geometry to the existence of a solution to the number theory behind Euler's criterion.

Title: On a Closed Formula for the Multiplicity of the Weights of the Lie Algebra $\mathfrak{sl}_4(\mathbb{C})$
Presenter: Gabriel Ngwe
Affiliation: Williams College
Abstract: Let $\mathfrak{g}$ be a semisimple Lie algebra and $\tau : \mathfrak{g} \to \mathfrak{gl}(V)$ be a finite-dimensional complex irreducible representation of $\mathfrak{g}$. A consequence of the theorem of highest weight is that τ is equivalent to a highest weight

representation with dominant integral highest weight λ, which we denote by $L(\lambda)$. It is also known that the underlying vector space V can be decomposed as a direct sum of subspaces V_μ, which are associated to the weights μ. Our question of interest is determining the dimension of each weight space V_μ in the decomposition of V. To compute this dimension we use Kostant's weight multiplicity formula, an alternating sum over a finite group involving Kostant's partition function. A major complication in using Kostant's weight multiplicity formula is that closed formulas for the partition function are not known in much generality. We present a closed formula for the partition function for the Lie algebra $\mathfrak{sl}_4(\mathbb{C})$ and present a computationally efficient method for finding the dimension of the weight spaces V_μ.

Title: The Colorability of Rational Tangles
Presenter: Dawn Paukner
Affiliation: University of Wisconsin Eau Claire
Abstract: Our research team has begun research on the colorability of rational tangles. The colorability of a rational tangle is an invariant of the tangle. We investigated the relationship between the colorability of the tangle and its rational number, which is a complete invariant of the tangle. We have proven formulas for the determinant for arbitrary but specific types of rational tangles. We also have investigated the relationship between the colorability of the tangle and its closures.

Title: A Combinatorial Problem on Finite Abelian Groups
Presenter: Darleen Perez-Lavin
Affiliation: University of Kentucky
Abstract: At the Midwestern conference on Group Theory and Number Theory in 1966, H Davenport presented the zero sum problem in the following manner. Let F be an algebraic number field and G be the class group of F. Then s(G) is the maximal number of prime ideals in the decomposition of an irreducible integer in F. In 1999, Chapman, Freeze and Smith define the strong Davenport constant and look at the set of equivalence classes of the minimal zero-sequences.

Title: Alternating Volume, a Hyperbolic Invariant of Knots
Presenter: Leslie Rodriguez
Affiliation: California State University Long Beach
Abstract: A knot is a loop in three-dimensional space. Alternating knots are a class of knots with useful geometric properties. Using methods originally due to Blair, we define the alternating volume of a knot to be the volume of an alternating link representation of the knot. We then extend results due to Lackenby to relate the Alternating Volume to twist number.

This project is supported in part by NSF grant DMS-1247679

Title: Topological complexity for driverless vehicles
Presenter: Ricky Salgado
Affiliation: Wilbur Wright College
Abstract: The topological complexity is a numerical invariant which measures the number of commands an autonomous robot needs in order to move in a space to perform a task.

To explain these ideas, we will walk through the various algebraic definitions and provide physical examples along the way. We calculate the topological complexity for various scenarios, starting with a single robot moving on a simple space, we will add on to this scenario towards a final case including more robots and more complicated spaces where they move.

Finally, we provide an example where two driverless vehicles move in a track joining seven colleges throughout Chicago. We determine the number of instructions as well as their content for this case study.

Title: Computing Commuting Partitions
Presenter: Carlos Samuels
Affiliation: CUNY Scholar Research Program Medgar Evers College CUNY
Abstract: A square matrix N is called nilpotent if $N^k = 0$ for some positive integer k. By the Jordan Normal Form Theorem, the conjugacy classes of nilpotent $n \times n$ matrices are in one-to-one correspondence with the partitions of n. We say that two partitions P and Q of n commute if there is a nilpotent matrix A of partition P and nilpotent matrix B of partition Q such that $AB = BA$. In this undergraduate research project we are using the open-source mathematics software system Sage to find partitions that commute with a partition $P = (u, u - r)$ where $u > r > 1$.

Let A be the generic matrix that commutes with a Jordan block matrix of type P. We have written a program that computes all the partitions that can be obtained by setting some of the variables in A equal to 0, and another program that computes all the partitions that can be obtained by letting the variables in A vary over all the values in a finite field.

Title: A Homotopy-Theoretic Approach to the Topological Tverberg Conjecture
Presenter: Bridget Schreiner
Affiliation: Wellesley College
Abstract: Let $r \geq 2$ and $d \geq 1$ be integers, let $N = (d+1)(r-1)$, and let Δ^N denote a standard N-simplex. The Topological Tverberg Conjecture states that for any continuous map $f : \Delta^N \to \mathbb{R}^d$, there are r pairwise disjoint faces $\sigma_1, \ldots, \sigma_r$ of Δ^N such that $f(\sigma_1) \cap \ldots \cap f(\sigma_r)$ is non-empty. F. Frick recently announced a counterexample to the conjecture for $d \geq 3r + 1$, when r is not a power of a prime. My poster will discuss an alternative analysis of this counterexample using the manifold calculus of functors, a technique that we hope will provide insight into the minimal counterexample and other related questions.

Title: Image-based data analysis via discrete Morse theory and persistent homology
Presenter: Christopher Szul
Affiliation: University of Illinois at Urbana-Champaign
Abstract: A key task of of data science is the ability to analyze big collections of data with the goal of understanding its behavior and possibly predicting patterns.Yet this type of inquiry traditionally requires a lot of experimental data, time, and financial resources. Discrete Morse Theory provides a toolkit for studying key qualitative properties of shapes. A team of Australian scientists has successfully applied this toolkit in order to extract information about digital images. We have adapted their ideas for use in the comparison of heat maps of certain data sets. Concretely, we applied these techniques to analyze water scarcity maps of certain countries (Kazakhstan) and are using it to predict water levels for the coming years.

Title: Kasteleyn Cokernels of Planar Bipartite Graphs
Presenter: Libby Taylor
Affiliation: Georgia Institute of Technology
Abstract: The Kasteleyn cokernel of a planar bipartite graph is a quotient group consisting of integer linear combination of its vertices modulo its signed bipartite adjacency matrix. In this result, we describe the relationship between the Kasteleyn cokernel of a graph and its set of perfect matchings. In a special subclass of planar bipartite graphs which are due to Kenyon, Propp, and Wilson, there exists an elegant algorithm producing a bijection between these two objects.

Title: Monomial Orders Uniquely Determined by Their Induced Orders
Presenter: Lana Vali
Affiliation: Amherst College
Abstract: It is known that in a polynomial ring with n variables, n ¿ 3, the classical monomial orders (lexicographic, graded lexicographic and reverse lexicographic) have the property that they are uniquely determined by their induced orders. Recently, it was proven that there is an infinite family of pairs of different monomial orders inducing the same orders. In this talk we present a general family of monomial orders, including a variation of

the reverse lexicographic order, that are uniquely determined by their induced orders.

Title: Using Representations of $\mathfrak{sl}_2(\mathbb{C})$ to Show Rank Unimodality
Presenter: Julianne Vega
Affiliation: University of Kentucky
Abstract: We will consider a combinatorial problem in terms of posets and detail a result by Robert Proctor, in which he uses representations of $\mathfrak{sl}_2(\mathbb{C})$, the lie algebra for special linear 2×2 matrices, to prove rank unimodality. We will do so by defining raising and lowering operators with weight vectors in a representation of $\mathfrak{sl}_2(\mathbb{C})$ and considering the modality of the resulting ladder.

Title: A Novel Definition of Bridge Number
Presenters: Roman Velazquez and Paul Villanueva
Affiliation: California State University, Long Beach
Abstract: We examine generating sets of the Wirtinger presentation of a knot group. We define the meridional coloring number to be the minimal number of generators required in a generating set of the Wirtinger presentation of a knot group. We then prove that the meridional coloring number is equal to the bridge number of the knot. We conclude with a computer-aided approach to finding the meridional coloring number, applications to Cappell and Shaneson's conjecture that the bridge number of a knot is equal to its meridional rank, and future work.

Title: Generalizing Zeckendorf's Theorem Via Bin Sequences
Presenter: Cameron Voigt
Affiliation: United States Military Academy at Westpoint
Abstract: Zeckendorf's Theorem states that any positive integer can be uniquely decomposed as a sum of non-consecutive Fibonacci numbers. In this project, we seek to generalize Zeckendorf's Theorem with a process involving varied bin sequencing. Using these bins and our legal limits, we can analyze Zeckendorf decompositions and determine if there is Gaussian behavior. We develop this framework to draw conclusions about Zeckendorf's Theorem and the greater analysis of integer decomposition.

Title: Exploring the Partially Ordered Abelian Group of Combinatorial Games
Presenter: Devon Vukovich
Affiliation: Moravian College
Abstract: Game Theory is the mathematical field focused on studying models representing strategic decision making situations of cooperation and competition. One branch of Game Theory looks to study combinatorial games in which at least two opponents alternate moves with complete knowledge of the game and the goal of being the player to make the last possible move. Interestingly, these games form a partially-ordered abelian group under addition. This poster explores how algebra arises in another field of mathematics and how it can be used to simplify the study and analysis of combinatorial games.

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