

# USTARS

underrepresented students in topology and algebra research symposium

**Florida Gulf Coast University**

Fort Myers, Florida

April 17-19 2015



# Agenda

## Friday, April 17

6:00-10:00pm **Arrival to USTARS 2015**  
Early Registration  
Location: Courtyard Marriott Gulf Coast Town Center

## Saturday, April 18

8:00-12:00pm **Registration**  
Lutgert Hall Atrium

9:00-9:30am **Welcome and Opening Remarks**  
Dr. Erik Insko, Organizer, Assistant Prof. FGCU Mathematics Dept.  
Dr. Richard Schnackenberg, Department Chair, FGCU Mathematics Dept.  
Dr. Robert Gregerson, Dean, FGCU College of Arts and Sciences  
Location: Marieb Hall 100

9:30-10:00am **Session I**  
*On Artin's conjecture*  
Luis Sordo Vieira, University of Kentucky  
Location: Lutgert Hall Room 2201

*Contact Structures Supported by Handlebodies*  
Marcos Ortiz, The University of Iowa  
Location: Lutgert Hall Room 2202

*Topological Complexity of Fiber Spaces*  
Kristin Duling, West Virginia University  
Location: Lutgert Hall Room 2209

10:15-10:45am **Session II**  
*Universal Deformation Rings and Semidihedral 2-groups*  
Roberto Soto, The University of Iowa  
Location: Lutgert Hall Room 2201

*Teichmüller Space of Complex Hyperbolic Manifolds*  
Gangotryi Sorcar, Binghamton University  
Location: Lutgert Hall Room 2202

*Peak Sets of Coxeter Groups of Classical Lie Type*  
Darleen Perez-Lavin, Florida Gulf Coast University  
Location: Lutgert Hall Room 2209

10:45-11:15am **Coffee Break**  
Location: Lutgert Hall Atrium

11:15-12:30pm **Invited Faculty Speaker**  
*Why do topologists care about category theory?*  
Dr. Angelica Osorno, Reed College  
Location: Marieb Hall 100

12:30-1:30pm **Lunch**  
Location: Lutgert Hall Atrium

## Saturday, April 18

### 1:30-2:00pm **Session III**

*On Classifying  $N$ -Posets*

Cory Colbert, University of Texas at Austin

Location: Lutgert Hall Room 2201

*Discrete Homotopies and the Fundamental Group*

Eleanor Abernethy, United States Military Academy, West Point

Location: Lutgert Hall Room 2202

*Collisions among Random Walks on a Graph*

Rade Stoisavljevic, Florida Gulf Coast University

Location: Lutgert Hall Room 2209

### 2:15-2:45pm **Session IV**

*Deformations of Incidence Algebras and Cohomology*

Gerard Koffi, The University of Iowa

Location: Lutgert Hall Room 2201

*Frobenius Algebras derived from the Kauffman bracket Skein algebra*

Nelson Abdiel Colon-Vargas, The University of Iowa

Location: Lutgert Hall Room 2202

*$W$ -graphs over non-commutative rings*

Alexander Diaz-Lopez, University of Notre Dame

Location: Lutgert Hall Room 2209

### 2:45-3:15pm **Session V**

*The geometric construction of KLR-algebras*

Mee Seong Im, University of Illinois

Location: Lutgert Hall Room 2201

*Regularity of Tor for Weakly Stable Ideals*

Katie Analdi, University of Notre Dame

Location: Lutgert Hall Room 2202

*Equivariant Commutative Ring Spectra*

Sebastian (Mychael) Sanchez, University of Illinois

Location: Lutgert Hall Room 2209

### 3:45-4:45pm **Poster Session and Coffee**

Location: Lutgert Hall Atrium

### 4:45-6:00pm **Informal Networking Time**

Location: Lutgert Hall Atrium

### 6:00-8:00pm **Symposium Banquet**

Location: Sugden Resort and Hospitality Center 114

## Sunday, April 19

9:30-10:00am

### **Session VI**

*Effective Conjugacy of Nilpotent Groups via Geometry*

Mark Pengitore, Purdue University

Location: Lutgert Hall Room 2201

*Contact Structures, Open Books, and Lens Spaces*

Camila Ramirez, The University of Iowa

Location: Lutgert Hall Room 2202

*On the ranks of  $Fi_{24}$*

Faryad Ali, Al Imam Mohammad Ibn Saud Islamic University

Location: Lutgert Hall Room 2209

10:20-11:20am

### **Distinguished Graduate Student**

*The many applications of Grobner bases*

Gabriel Sosa, Purdue University

Location: Marieb Hall 100

11:30am- 12:30pm

### **Networking Lunch and Symposium Closing**

Location: Sugden Resort and Hospitality Center 114

#### **Panelists:**

Dr. Jocelyn Bell, United States Military Academy, West Point

Dr. Aba Mbirika, University of Wisconsin- Eau Claire

Dr. Candice Price, United States Military Academy, West Point

Mr. Roberto Soto, The University of Iowa

**Moderator:** Dr. Syvillia Averett, Central State University

## *Presentation Abstracts*

**Title:** Discrete Homotopies and the Fundamental Group

**Presenter:** Eleanor Abernethy

**Affiliation:** United States Military Academy, West Point

**Abstract:** This is a topic I am learning about and would like to take the opportunity to present some work of my advisor, Conrad Plaut, which generalizes and strengthens a theorem of Gromov's on compact Riemannian manifolds, a consequence of which is a fundamental group finiteness theorem. I would also like to present a conjecture which follows from this work.

**Title:** On the ranks of  $Fi'_{24}$

**Presenter:** Faryad Ali

**Affiliation:** Al Imam Mohammad Ibn Saud Islamic University

**Abstract:** If  $G$  is a finite group and  $X$  a conjugacy class of elements of  $G$ , then we define  $rank(G:X)$  to be the minimum number of elements of  $X$  generating  $G$ . In the present talk, we determine the  $rank(Fi'_{24}:X)$ , where  $X$  is any conjugacy class of  $Fi'_{24}$ .

**Title:** Regularity of Tor for Weakly Stable Ideals

**Presenter:** Katie Ansaldo

**Affiliation:** University of Notre Dame

**Abstract:** Eisenbud, Huneke, and Ulrich proved a bound for the regularity of  $\text{Tor}_i^R(R/I, R/J)$  provided that the dimension of  $\text{Tor}_1^R(R/I, R/J) \leq 1$  but there are examples where the bound does not hold in general. In this talk, we show the same bound holds when  $I$  and  $J$  are weakly stable ideals in a polynomial ring over a field. (Joint work with N. Clarke and L. Ferraro)

**Title:** On Classifying N-Posets

**Presenter:** Cory Colbert

**Affiliation:** University of Texas at Austin

**Abstract:** We say a partially ordered set  $U$  is an "N-Poset" if it is order isomorphic to the spectrum of a Noetherian ring, where the order relation in the spectrum is inclusion among prime ideals. Determining which posets are N-Posets is a very difficult problem that has led to decades of interesting research. We will briefly survey basic concepts and discuss results by some of the pioneers of this fascinating area.

**Title:** Frobenius Algebras derived from the Kauffman bracket Skein algebra

**Presenter:** Nelson Abdiel Colon-Vargas

**Affiliation:** The University of Iowa

**Abstract:** We will introduce the skein modules of the surfaces,  $\sum i, j \quad (i, j) \in \{(0, 2), (0, 3), (1, 0), (1, 1)\}$  at  $2N$ -th roots of unity where  $N \geq 3$  is an odd counting number and construct Frobenius algebras from them.

**Title:**  $W$ -graphs over non-commutative rings

**Presenter:** Alexander Diaz-Lopez

**Affiliation:** University of Notre Dame

**Abstract:** Given a Coxeter system  $(W, S)$ , a  $W$ -graph is a graph, together with additional information that encodes a representation of the Hecke algebra associated to  $W$ , called the  $\tau$  representation. Given an Iwahori-Hecke datum  $D$  I define pre- $D$ -graphs, a generalization of  $W$ -graphs, and construct representations of Hecke algebras on quotient path algebras (QPA) over suitable quivers. When the pre- $D$ -graph is given by a  $W$ -graph, I discuss the relationship between these representations and the  $\tau$  representations. Several important examples are discussed, including the QPA associated to the universal pre- $D$ -graph, as well as cases where the obtained QPA is isomorphic to the ideal of the asymptotic Hecke algebra associated to the reflection representation.

**Title:** Topological Complexity of Fiber Spaces

**Presenter:** Kristin Duling

**Affiliation:** West Virginia University

**Abstract:** The topological complexity of a configuration space, denoted  $TC(X)$ , is a homotopy invariant which measures the complexity of path planning on a space  $X$ . Previous results provide an upper bound for the topological complexity of the total space  $E$  of a fiber space  $(E, p, B, F)$  based on the topological complexity of the fiber, and the Lusternik-Schnirelmann category of the product space of the base with itself,  $cat(B \times B)$ . We demonstrate an upper bound for the topological complexity of the total space which replaces  $cat(B \times B)$  in the previous result with the topological complexity of the base space. A natural inequality states that  $TC(B) \leq cat(B \times B)$  for any space  $B$ , so for some base spaces, such as  $S^1$  this provides a smaller upper bound. In particular, this result can be applied to the Klein bottle and combine with a lower bound to show that  $TC(K) = 4$ .

**Title:** The geometric construction of KLR-algebras

**Presenter:** Mee Seong Im

**Affiliation:** University of Illinois

**Abstract:** The construction of quiver Hecke algebras arose while attempting to categorify quantum groups. Since their discovery, new and interesting mathematics have been developed by mathematicians, including 2-categories called 2-Kac-Moody algebras. These 2-categories categorify Lusztig's idempotent version of the quantized enveloping algebra of a Lie algebra, with an application in 4-dimensional TQFTs. I will begin with the geometric construction of quiver Hecke algebras as given by Rouquier, and independently by Varagnolo-Vasserot, and end with a discussion on some open problems related to KLR-algebras.

**Title:** Deformations of Incidence Algebras and Cohomology

**Presenter:** Gerard Koffi

**Affiliation:** The University of Iowa

**Abstract:** Incidence algebras were introduced in the 1960's by Gian-Carlo Rota as a way to study combinatorial problems but it became later apparent that such algebras were interesting object to study on their own; they have application in geometry and in topology. Examples of Incidence algebras include such algebras as the product of copies of a ring  $R$  and the upper triangular matrices over  $R$ . In this talk, we define deformations of incidence algebras and show that such deformations relate to cohomology. Using distributive modules and quivers, we describe all basic algebras that are deformations of incidence algebras.

**Title:** Contact Structures Supported by Handlebodies

**Presenter:** Marcos Ortiz

**Affiliation:** The University of Iowa

**Abstract:** Using a mixture of topological and combinatorial techniques it is possible to classify tight contact structures on handlebodies. The challenge is to find ways to understand the connections between the infinite possible configurations and divide them into equivalence classes. Using tools in convex decomposition theory developed by Ko Honda, the classification can be completed for genus-2 handlebodies and gives a solid foundation for future results in handlebodies of higher genus.

**Title:** Why do topologists care about category theory?

**Presenter:** Dr. Angelica Osorno \*\*

**Affiliation:** Reed College

**Abstract:** The study of category theory was started by Eilenberg and MacLane, in their effort to codify the axioms for homology. Category theory provides a language to express the different structures that we see in topology, and in most of mathematics. Categories also play another role in algebraic topology. Via the classifying space construction, topologists use categories to build spaces whose topology encodes the algebraic structure of the category. This construction is a fruitful way of producing important examples of spaces used in algebraic topology. In this talk we will describe how this process works, starting from classic examples and ending with some recent work.

**Title:** Effective Conjugacy of Nilpotent Groups via Geometry

**Presenter:** Mark Pengitore

**Affiliation:** Purdue University

**Abstract:** We give a solution to the conjugacy problem for finitely generated groups via homomorphisms onto finite groups. We further analyze the complexity of this solution by introducing a function which maximizes the order of the minimal finite group that separates a pair of non-conjugate elements as we vary over the  $n$ -ball. We provide asymptotic bounds for this function for finitely generated nilpotent groups through the use of a simply connected, connected nilpotent Lie group, which we can naturally associate to any finitely generated torsion-free nilpotent group.

**Title:** Peak Sets of Coxeter Groups of Classical Lie Type

**Presenter:** Darleen Perez-Lavin

**Affiliation:** Florida Gulf Coast University

**Abstract:** We say that a permutation  $\pi = \pi_1\pi_2\cdots\pi_n$  in the symmetric group  $S_n$  has a *peak* at index  $i$  if  $\pi_{i-1} < \pi_i > \pi_{i+1}$  and we let  $P(\pi) = \{i \mid i \text{ is a peak of } \pi\}$ . Given a set  $S$  of positive integers we let  $P(S; n)$  denote the subset of  $\mathfrak{S}_n$  consisting of all permutations  $\pi$ , where  $P(\pi) = S$ . In 2013, Billey, Burdzy and Sagan proved that  $|P(S; n)| = p(n)2^{n-|S|-1}$ , where  $p(n)$  is a polynomial of degree  $\max(S) - 1$ . In 2014, Casto et, al. considered the Coxeter group of type  $B$  as the group of signed permutations on  $n$  letters and showed that  $|P_B(S; n)| = p(n)2^{2n-|S|-1}$ . We partition the set  $P(S; n)$  into subsets of elements which end with an ascent or a descent and provide recursive formulas for the cardinalities of these subsets. We then embed the Coxeter groups of Lie type  $B_n$  and  $D_n$  into the symmetric group  $\mathfrak{S}_{2n}$  and use the partitioning of  $P(S; n)$  to describe and enumerate the sets  $\widehat{P}_B(S; n)$ ,  $\widehat{P}_B(S \cup \{n\}; n)$ ,  $\widehat{P}_D(S; n)$  and  $\widehat{P}_D(S \cup \{n\}; n)$ . Furthermore, these results lead to a collection of interesting identities as special cases of our enumerative formulas for  $|\widehat{P}_B(S; n)|$  and  $|\widehat{P}_D(S; n)|$ .

**Title:** Contact Structures, Open Books, and Lens Spaces

**Presenter:** Camila Ramirez

**Affiliation:** The University of Iowa

**Abstract:** A contact structure on a  $(2n + 1)$  differentiable manifold  $M$ , is a hyperplane field which can be written locally as the kernel of a 1-form, call it  $a$ , with the property that  $a^d a$  is a volume form. We explore the relations between contact structures and open books on  $M$  and explain how we can detect a virtually overtwisted tight contact structure on Lens Spaces using open book decompositions.

**Title:** Equivariant Commutative Ring Spectra

**Presenter:** Sebastian (Mychael) Sanchez

**Affiliation:** University of Illinois

**Abstract:** In non-equivariant homotopy theory, a commutative ring spectrum is a spectrum with a multiplication that is commutative up to coherent homotopy. In the presence of a group action, each family of subgroups determines a distinct collection of equivariant commutative ring spectra. Of particular interest is the family consisting of only the trivial subgroup and the family consisting of all subgroups. These families determine the notion of an  $E_\infty$  commutative ring spectrum and  $E_\infty^G$  commutative ring spectrum. In this talk, I'll discuss each of these notions of commutative ring and several problems that arise when comparing them.

**Title:** Teichmüller Space of Complex Hyperbolic Manifolds

**Presenter:** Gangotryi Sorcar

**Affiliation:** Binghamton University

**Abstract:** The talk is on results concerning complex hyperbolic manifolds  $M$ . We prove that  $T^{<0}(M)$  is non contractible by constructing a non trivial element in  $\pi_1(T^{<0}(M))$ , where  $T^{<0}(M)$  denotes the Teichmüller space of all negatively curved Riemannian metrics on  $M$ , which is the quotient space of the space of all negatively curved Riemannian metrics on  $M$  modulo the space of all isotopies of  $M$  that are homotopic to the identity.

**Title:** On Artin's conjecture

**Presenter:** Luis Sordo Vieira

**Affiliation:** University of Kentucky

**Abstract:** Artin conjectured that any homogeneous polynomial over the  $p$ -adics of degree  $d$  in more than  $d^2$  variables has a non trivial zero. Terjanian disproved the conjecture. In the case of diagonal forms, the result holds by a theorem of Davenport and Lewis. The result for diagonal forms over algebraic extensions of the  $p$ -adics is unknown. We investigate some of the known results.

**Title:** The many applications of Grobner bases

**Presenter:** Gabriel Sosa\*

**Affiliation:** Purdue University

**Abstract:** "The theory of Gröbner bases was originally devised as a way to solve the ideal membership problem in polynomial rings over a field in more than one variable. Since then it has had a profound impact on commutative algebra by integrating the use of computers, computational tools and programs, like Macaulay 2 and CoCoA, in the study, and solution, of problems in commutative algebra. Allowing for the calculation of solutions of systems polynomial equations, syzygies and free resolutions.

It is not surprising then that sufficient conditions for certain algebraic properties of toric rings and monomial ideals, like normality, being Cohen-Macaulay or Koszul can be established in terms of features of Grobner bases and/or initial ideals. This particular application has proven particularly useful in the study of combinatorial objects such as convex polytopes, posets and graphs.

In this talk we will provide an overall introduction to the computational and theoretical power of Grobner bases.

**Title:** Universal Deformation Rings and Semidihedral 2-groups

**Presenter:** Roberto Soto

**Affiliation:** The University of Iowa

**Abstract:** Let  $k$  be an algebraically closed field of characteristic 2, let  $SD$  be a semidihedral 2-group of order at least 16, and let  $V$  be an indecomposable  $kSD$ -module. By work of Bleher and Chinburg,  $V$  has a universal deformation ring  $R(SD, V)$  when the stable endomorphism ring  $\underline{End}_{kSD}(V, V)$  is isomorphic to  $k$ . In this talk we use work of Carlson and Thévenaz to classify all such  $kSD$ -modules, and we discuss how to determine their universal deformation rings.

**Title:** Collisions among Random Walks on a Graph

**Presenter:** Rade Stoisavljevic

**Affiliation:** Florida Gulf Coast University

**Abstract:** Inspired by the Cops and Robbers game introduced by Nowakowski and Winkler. My variation of the game includes two players, "drunken" cop and robber who are taking random walks on a graph. I computed the probabilities that the robber will stay free (will not collide with the cop) on Complete graphs, Complete Bipartite graphs, Friendship graphs, Windmills and Cycles. I computed the recursive and closed formulas.

\*\* Invited Faculty Speaker

\* Distinguished Graduate Student

## *Poster Abstracts*

**Title:** Topological analysis of brain data

**Presenter:** Leyda Almodovar Velazquez

**Affiliation:** The University of Iowa

**Abstract:** Topological data analysis is a relatively new area that uses several disciplines in conjunction such as topology, statistics and computational geometry. The idea behind topological data analysis is to describe the shape of data by recovering the topology of the sampled space. Also, it is useful to find topological attributes that persist in the data, helping us gain a better understanding of how different properties of the data interact. Data was collected from fMRI experiments with 96 subjects between the ages of 6 and 18, some of them predisposed to Huntingtons disease. Data analysis is performed via different topological approaches including clustering and persistent homology with the goal of identifying whole networks of points in the brain. The main purpose of this work is to compare the structure of brain networks of healthy subjects versus subjects predisposed to Huntingtons disease.

**Title:** Colorability of n-Whitehead Doubles of Knots

**Presenter:** Danielle Brushaber

**Affiliation:** University of Wisconsin- Eau Claire

**Abstract:** Our research team is looking at the colorability of the n-Whitehead doubles of knots. Colorability is an invariant that helps us classify knots. Using linear algebra, we create a matrix for each knot, and the minor determinant of this matrix determines the knot's colorability. We found a pattern relating n to the colorability of certain knots and will present the evidence supporting this. We are also investigating a potential pattern between the colorability of the original knot and the colorability of the whitehead double where  $n=1$ .

**Title:** Bounding k-distance domination numbers of grids

**Presenter:** Armando Grez

**Affiliation:** Florida Gulf Coast University

**Abstract:** In his 1992 Ph.D. thesis Chang identified an efficient way to dominate  $mn$  grid graphs and conjectured that his construction gives the most efficient dominating sets for relatively large grids. In 2011 Goncalves, Pinlou, Rao, and Thomasse proved Changs conjecture, establishing a closed formula for the domination number of a grid. In March 2013 Fata, Smith and Sundaram established upper bounds for the k-distance domination numbers of grid graphs by generalizing Changs construction of dominating sets to k-distance dominating sets. In this paper we improve the upper bounds established by Fata, Smith, and Sundaram for the k-distance domination numbers of grids.

(This is joint work with Michael Farina)

**Title:** Patrolling Dominating Sets of Graphs

**Presenter:** Gabriel Guillen and Skyrie Louissaint

**Affiliation:** Florida Gulf Coast University

**Abstract:** Due to its many real world applications, there have been over 2000 papers published on domination theory in graphs. Traditionally the elements of a dominating set have remained stationary, but in patrolling domination theory, we require the dominating elements to "patrol" a set of vertices. In this poster, we show that the patrolling domination number of a graph is bounded below by its domination number and bounded above by twice its domination number. We then show that both of these bounds are tight by identifying families of graphs for which the lower bound is an equality and families of graphs for which the upper bound is an equality. Then we analyze domination numbers of grid graphs.

**Title:** Stability Analysis of Goryachev-Chaplygin top and its topology

**Presenter:** Fariba Khoshnasib

**Affiliation:** University of Texas at Dallas

**Abstract:** This is a research built upon special, integrable cases of rigid body motion. The most celebrated example is the top established by S. Kowalevski. The differential equations of motion of such systems are Integrable Hamiltonian ones. The Goryachev-Chaplygin case is the focus of our interest. This system is integrable on a fixed level of one of the geometric integrals, where one can obtain an additional, fourth integral. By the Liouville-Arnold theorem, we conclude that this system is completely integrable and the common level set of the first integrals  $H$  and  $F$  is a smooth manifold that is invariant under the phase flow of the system. Also, we know every connected component of the common level set (symplectic leave) is diffeomorphic to a two-dimensional torus. The phase space here is a six-dimensional space with Lie-Poisson bracket. The bifurcation diagram of such completely integrable system is the region of possible motion on the plane of first integrals together with the image of the critical set. In the case of Goryachev-Chaplygin top, one of the connected components on the integral manifold is an ordinary Liouville torus and lacks any critical points. To overcome this issue, using Andoyer variables, the invariant tori are constructed and singular leaves stability behavior are analyzed and compared to the traditional stability analysis using a parameter.

**Title:** Singularities of Schubert Varieties Corresponding to Coxeter Elements

**Presenter:** Christie Mauretour

**Affiliation:** Florida Gulf Coast University

**Abstract:** While studying the intersection theory of the Peterson Variety, Insko and Tymoczko asked the question “When is the opposite Schubert variety corresponding to a Coxeter Element singular?” This is a preliminary report that answers this question in certain Lie types.

**Title:** A look at some relationships between knots and graphs.

**Presenter:** Sooji Park

**Affiliation:** United States Military Academy, West Point

**Abstract:** I am combining my interests in both knot theory and graph theory by studying their relationships. I hope to discover an efficient way to label or color graphs and corresponding knots by studying the correlation. First part discusses planar graphs and its corresponding alternating knots by polynomial expression. There is a correlation between the dichromatic polynomial of a graph and the square bracket polynomial of the corresponding alternating knot. This poster shows examples of the relationship of complete graphs, bipartite graphs, and trees. The second part articulates nonplanar graphs as intrinsically knotted. Based on the theorem that all planar graphs with  $E \geq 4V - 17$ , where  $E$  is number of edges and  $V$  is number of vertices, are intrinsically knotted, I show different bipartite graphs that prove the theorem. I will also show a few examples of intrinsically knotted complete graphs.

*Special thanks to the following USTARS sponsors, partners, and supporters:*

The National Science Foundation  
Florida Gulf Coast University  
The FGCU Mathematics Department  
The FGCU Whitaker Center  
The FGCU Office of Research and Graduate Studies  
The FGCU College of Arts and Sciences  
Algenol, LLC  
The University of California, Berkeley Mathematics Department  
The FGCU Mathematics Club  
The USTARS Advisory Board  
Dr. David Eisenbud  
Dr. Derek Buzasi  
Kathy McElroy