USTARS

underrepresented students in topology and algebra research symposium

Iowa State University
Ames, Iowa
April 26 - April 28, 2019

IOWA STATE UNIVERSITY

Agenda

Friday, April 26, 2019

6:00am -10:00pm **Arrival to USTARS**
Location: Gateway Hotel and Conference Center at Iowa State University

Saturday, April 27, 2019

8:00am -12:00pm **Registration**
Location: Carver 205

9:30am -9:45am **Welcome and Opening Remarks**
Dr. Candice Price, University of San Diego
Location: Carver 205

10:00am - 10:30am **Session I**
Positivity of Knots
Elaina Aceves, University of Iowa
Location: Carver 204

Generators of Koszul Homology with Coefficients in a $\underline{g}$-Weak Complete Intersection Module
Rachel Diethorn, Syracuse University
Location: Carver 268

10:40 -11:10am **Session II**
CV-Modules and Generalized Demazure Modules
Justin Davis, University of California, Riverside
Location: Carver 202

Plus-Minus Davenport Constant
Darleen Perez-Lavin, University of Kentucky
Location: Carver 204

Elementary theories and embeddings of groups
Christopher Perez, University of Illinois at Chicago
Location: Carver 268

11:20-11:45am **Session III: Informal Networking**

Saturday, April 28, 2019

11:45-1:45pm **Lunch and Mentoring Panel**
Location: Carver 205
Panelists:
Robyn Brooks, Tulane University
Dr. Gerard Koffi, University of Texas at Arlington
Dr. Alexander Barrios, Carleton College
Dr. Johanna Hennig, IDA Center for Communications and Computing
Moderator: Dr. Javier Ronquillo, Grand Valley State University

2:00-3:30pm **Invited Faculty Speaker**
Group Inverses of Matrices, Sign Patterns and Applications
Dr. Minerva Catral, Xavier University
Location: Carver 205

3:45-5:00pm **Poster Session and Informal Networking**
Location: Carver 205

5:10-5:30pm **Group Photo**
Location: Carver 205

5:30-6:30pm **Distinguished Graduate Speaker**
Trivalent Spatial Graphs and Niebrzydowski Algebras
Sherilyn Tamagawa, University of California, Santa Barbara
Location: Carver 205

7:00-8:30pm **Symposium Banquet**
Location: Garden Room, Gateway Hotel

Sunday, April 28, 2018

9:30-10:00am **Session IV**
Holomorphic Differentials of a Projective Curve
Nicholas Camacho, The University of Iowa
Location: Carver 202

Bi-Orderability of Knot Groups
Jonathan Johnson, University of Texas, Austin
Location: Carver 204

10:10am-10:40am **Session V**
Defining Equations of Rees Algebras
Whitney Liske, University of Notre Dame
Location: Carver 202

Techniques for determining the homotopy type of matching complexes
Julianne Vega, University of Kentucky
Location: Carver 204

10:50 - 11:50am **Distinguished Graduate Speaker**
Blow-up constructions and the defining ideal of Rees algebras
Alessandra Constantini, Purdue University
Location: Carver 205

12:00-12:45pm **Lunch and Symposium Closing**
Location: Carver 205

USTARS 2018 Invited Faculty Speaker

Dr. Minerva Catral, Xavier University

Minnie was born and raised in Manila, Philippines. She received a B.S. in mathematics from the University of the Philippines, an M.S. in mathematics from the University of Washington, and a Ph.D. in mathematics from the University of Connecticut. After completing her Ph.D. in 2005, Minnie held several academic positions, including a senior lecturer position at Drexel University in Philadelphia, a post-doctoral fellowship at the University of Victoria (Canada) and a post-doctoral research associate position at Iowa State University. She is currently an associate professor at Xavier University in Cincinnati, Ohio. Minnie's teaching and research interests are in matrix theory and applied linear algebra.

USTARS 2019 Distinguished Graduate Students

USTARS 2019 Distinguished Graduate Student in Topology

Sherilyn Tamagawa, University of California - Santa Barbara

Sheri was born and raised in Hawaii, and first fell in love with math in the sixth grade. After high school, she attended Scripps College in Claremont, CA where she majored in math and wrote a senior thesis on knot theory.

Immediately after undergrad, she went to UC Santa Barbara for graduate school, and is currently a (graduating!) fifth-year. With her advisor Stephen Bigelow, she studied more knot theory and also started studying planar algebras. Apart from her advisor, Sheri has studied other fun low-dimensional things like trivalent spatial graphs and pseudoknots. (You aren't intended to know what those mean off hand, but she encourages you to Google them.) She also discovered that her favorite part of academia is teaching.

Sheri is very excited to be attending USTARS for the third time! She hopes that everyone feels as welcome and supported at USTARS as she has.

Outside of math, she considers herself an indoor person. Her hobbies include knitting, baking, and binge watching crime shows.

USTARS 2019 Distinguished Graduate Student in Algebra

Alessandra Costantini, Purdue University

Alessandra is a Ph.D. student at Purdue University, working in commutative algebra under the direction of Prof. Bernd Ulrich. My passion for math started in my hometown, L'Aquila, in Italy. While working on my bachelors and masters degrees at the University of L'Aquila, I was particularly fascinated by the topics that would create connections between different fields of math. That's how commutative algebra - with its connections to algebraic geometry, number theory and combinatorics, got my heart!

Besides learning lots of math and enjoying teaching lots of courses, at Purdue I have also learned how to dance Latin dances like salsa and bachata, and to cook and bake healthy yet delicious food.

Presentation Abstracts

Title: Positivity of Knots
Presenter: Elaina Aceves
Affiliation: University of Iowa
Abstract: In Jesse Hamer's thesis, he was able to classify the positivity of all knots up to twelve crossings except for two. In this talk, I will demonstrate my attempts to classify the two remaining knots.

Title: Skein Modules and the Thickened Four-Holed Sphere
Presenter: Rhea Palak Bakshi
Affiliation: The George Washington University
Abstract: In 1987 Jozef Przytycki introduced skein modules as a way to extend the knot polynomials of the 1980's to knots and links in arbitrary 3-manifolds. Since their introduction skein modules have become central to the theory of 3-manifolds. In 1997, Frohman and Gelca established a compact product-to-sum formula for the Kauffman bracket skein algebra of the torus times the interval. We try to discover a similar formula for the multiplication of curves in the thickened sphere with four holes and I will present some of our results to this end. This is joint work with Sujoy Mukherjee, Jozef Przytycki, Marithania Silvero and Xiao Wang.

Title: Holomorphic Differentials of a Projective Curve
Presenter: Nicholas Camacho
Affiliation: University of Iowa
Abstract: This talk will connect algebraic geometry and representation theory. We will discuss projective curves, and then define the sheaf of relative differentials on a projective curve. The global sections of this sheaf are called holomorphic differentials, and these form a vector space. Letting a group act on the projective curve, we try to understand the representation theory of the space holomorphic differentials.

Title: Group Inverses of Matrices, Sign Patterns and Applications
Presenter: Dr. Minerva Catral**
Affiliation: Insert School
Abstract: The role of the group generalized inverse in the study of finite Markov chains was described and highlighted in a seminal paper by Meyer. In my Ph.D. thesis, I explored the connections between group inverses, Markov chains, and mean first passage times. An overview of this work will be presented, followed by a discussion of subsequent research topics pursued post-Ph.D., including sign patterns and related topics in combinatorial matrix theory.

Title: Blow-up constructions and the defining ideal of Rees algebras
Presenter: Alessandra Costantini *
Affiliation: Purdue University
Abstract: Rees algebras of ideals and modules arise in Commutative Algebra and Algebraic Geometry in connection with blow-up constructions, in order to study singularities of algebraic varieties. In this talk, after reviewing the geometric construction of blow-ups, I will describe the problem of determining the defining ideal of the Rees algebra, i.e. a presentation in terms of generators and relations. This is a difficult problem in general, but becomes easier to treat in case the ideal or module has a nice presentation matrix. This work is a generalization of a result by Jacob Boswell and Vivek Mukundan.

Title: CV-Modules and Generalized Demazure Modules
Presenter: Justin Davis
Affiliation: University of California, Riverside
Abstract: In 2013 V. Chari and R. Venkatesh introduced a family of modules for the current algebra $\mathfrak{g} \otimes \mathbb{C}[t]$, for a semi simple Lie algebra $\mathfrak{g}$ indexed by n-tuples of partitions, where n is the rank of $\mathfrak{g}$. In this talk I will discuss some connections between these "CV-Modules" and generalized Demazure modules, which are submodules in the tensor product of local Weyl modules generated by the tensor product of highest weight vectors.

Title: Generators of Koszul Homology with Coefficients in a $\underline{g}$-Weak Complete Intersection Module
Presenter: Rachel Diethorn
Affiliation: Syracuse University
Abstract: Understanding the homology of the Koszul complex on a minimal set of generators for the maximal ideal in a local ring is often the key to discovering properties of the local ring itself. Similarly, understanding Koszul homology of any sequence of elements, $g_1, ..., g_s$ in a local ring R can give useful information about the quotient $R/(g_1, ..., g_s)$ as an R-module. In this talk, I will give explicit formulas for the generators of Koszul homology with coefficients in what we will call a $(\underline{g})$-weak complete intersection module. This generalizes work of Herzog and of Corso, Goto, Huneke, Polini, and Ulrich. I will begin by defining $(\underline{g})$-weak complete intersection modules and providing some background on the topic of Koszul homology. I will then briefly discuss the construction of such formulas. If time permits, I will highlight an application of these formulas.

Title: Bi-Orderability of Knot Groups
Presenter: Jonathan Johnson
Affiliation: University of Texas - Austin
Abstract: The orderability of 3-manifold groups has become a topic of interest in the last couple of decades. Most literature is concerned with relating the left-orderability of rational homology spheres to other 3-manifold properties. Unfortunately, the fundamental groups of complements of knots in S^3 are always left-orderable(Boyer-Rolfsen-Wiest). However, the bi-orderability of these groups is an interesting subject to study. In this talk, we will survey many of the result pertaining to the bi-orderability of knot groups.

Title: Defining Equations of Rees Algebras
Presenter: Whitney Liske
Affiliation: University of Notre Dame
Abstract: Let $R = k[x_1, \ldots, x_d]$ be a polynomial ring in d variables over a field k. Let $m = (x_1, \ldots, x_d)$ be the maximal homogenous ideal of R. Let I be a Gorenstein ideal generated by all the generators of m^2 except for one. For each fixed d these ideals are all equivalent, up to change of coordinates. The goal is to compute the defining equations of the special fiber ring and the Rees ring of these ideals. To compute the Rees ring, we study the Jacobian dual and the defining equations of the special fiber ring of m^2.

Title: Invariant of set-valued inverse limits
Presenter: Faruq Mena
Affiliation: Missouri University of Science & Technology and Soran University
Abstract: We will discuss what topological properties are preserved under inverse limit operation with set valued, upper semi-continuous functions. In particular we find conditions for the functions that preserve local connectedness, arc-like continua, tree like continua, hereditary decomposability, heriditary indecomposability, and others.

Title: Elementary theories and embeddings of groups
Presenter: Christopher Perez
Affiliation: University of Illinois at Chicago
Abstract: The *elementary theory* of a group G is the set $\mathrm{Th}(G)$ of all first-order sentences in the language of groups which are valid over G. Two groups G and H are said to be *elementarily equivalent* if $\mathrm{Th}(G) = \mathrm{Th}(H)$. A concept closely related to elementary equivalence is that of elementary embeddings: Let H be a subgroup of a group G. The inclusion of H into G is an *elementary embedding* if for any first-order formula $\varphi(x_1, \ldots, x_n)$ with $n \geq 0$ variables and any $(h_1, \ldots, h_n) \in H^n$, $\varphi(h_1, \ldots, h_n)$ is valid over H if and only if it is valid over G. In particular, $\mathrm{Th}(G) = \mathrm{Th}(H)$.

Zlil Sela proved in 2009 that any finitely generated group which is elementarily equivalent to a torsion-free hyperbolic group is necessarily hyperbolic as well, and in 2012 Chloé Perin proved that torsion-free hyperbolic groups have the structure of a *hyperbolic tower* over their elementarily embedded subgroups. In this talk we will discuss these results and recent work to generalize these results to toral relatively hyperbolic groups.

Title: Plus-Minus Davenport Constant
Presenter: Darleen Perez-Lavin
Affiliation: University of Kentucky
Abstract: Let G be a finite abelian group, written additively. The Davenport constant $D(G)$ is the smallest positive number s such that for for any set $\{g_1, g_2, \ldots, g_s\}$ of s elements in G, with repetition allowed, there exists a subset $\{g_{i_1}, g_{i_2}, \ldots, g_{i_t}\}$ such that $g_{i_1} + g_{i_2} + \cdots + g_{i_t} = 0$. The plus-minus Davenport constant, $D_{\pm}(G)$, is defined similarly but instead we only require that $g_{i_1} \pm g_{i_2} \pm \cdots \pm g_{i_t} = 0$. In this talk, we study the best known estimates for $D_{\pm}(G)$ when $G = C_3^r \oplus C_5^s$ and $G = C_7^r$.

Title: Trivalent Spatial Graphs and Niebrzydowski Algebras
Presenter: Sherilyn Tamagawa *
Affiliation: University of California, Santa Barbara
Abstract: In this talk, we will introduce Niebrzydowski algebras, algebraic structures with a ternary operation and a partially defined multiplication, whose axioms are motivated by the Reidemeister moves for Y-oriented trivalent spatial graphs and handlebody-links. We will also show that the counting invariant distinguishes some trivalent spatial graphs and handlebody-links.

Title: Techniques for determining the homotopy type of matching complexes
Presenter: Julianne Vega
Affiliation: University of Kentucky
Abstract: The study of matching complexes dates as far back as 1992 and has led to interesting applications. For example, the matching complex of $K_{m,n}$ has aided in the analysis of Tits coset complexes and been applied to computational geometry. While the study of the connectivity of these matching complexes has been rich, it is difficult to find the homotopy type of such complexes. In this talk, we will introduce techniques to determine the homotopy type of matching complexes which involves looking at the effects of graph operations on the matching complex.

** Invited Faculty Speaker
* Distinguished Graduate Student

Poster Abstracts

Title: Generalized (σ, τ)-derivations on prime near-rings
Presenter: Gbrel Albkwre
Affiliation: West Virginia University
Abstract: In 2005, Ö. Gölbafsii introduced a method to prove the commutativity in prime near-rings with generalized (σ, τ)-derivations, but his proof was wrong because it was assumed both a right and left distributive simultaneously. To achieve the complete and correct proof , we developed one of the lemma was given by Howard E. Bell, 2008, and more restrictions about the derivation.

Title: SL(2,Z) Action on Some Quadratic Differentials
Presenter: Paige Helms
Affiliation: University of California, Riverside
Abstract: The purpose of this research is to establish a lower bound for the number of orbits of the $SL_2(\mathbb{Z})$ action on the space of quadratic differentials of a genus g surface, which can be identified with the cotangent bundle to a point in Teichmüller space $\mathcal{T}_g$. We accomplish this through an algebraic interpretation of a pair of minimally intersecting curves that fill a surface Σ_g of genus g, where for $g > 2$, the minimal intersection number is $i(\alpha, \beta) = 2g - 1$. Such a pair of curves can be visualized on a square-tiling of a surface Σ_g that also carries the structure of an *origami*. We call this (α, β) pair with the origami a MIFPO. This interpretation gives us a way to examine the action of $SL_2\mathbb{Z}$ on a given surface by calculating its monodromy group, whose type gives us a lower bound for the number of orbits. So far, we have shown the existence of at least two orbits for $n = 5, n = 7, n = 9$, and $n = 11$.

Title: Combinatorics of k-Farey Graphs
Presenter: Miguel Lopez and Zoe Riell
Affiliation: Boston University
Abstract: With an eye towards studying curve systems on low-complexity surfaces, we introduce and analyze the k-Farey graphs $\mathcal{F}_k$ and $\mathcal{F}_{\leqslant k}$, two natural variants of the Farey graph $\mathcal{F}$ in which we relax the edge condition to indicate intersection number $= k$ or $\leq k$, respectively.
The former, $\mathcal{F}_k$, is disconnected when $k > 1$. In fact, we find that the number of connected components is infinite if and only if k is not a prime power. Moreover, we find that each component of $\mathcal{F}_k$ is an infinite-valence tree whenever k is even, and $\mathrm{Aut}(\mathcal{F}_k)$ is uncountable for $k > 1$.
As for $\mathcal{F}_{\leqslant k}$, Agol obtained an upper bound of $1 + \min\{p : p \text{ is a prime} > k\}$ for both chromatic and clique numbers, and observed that this is an equality when k is either one or two less than a prime. We add to this list the values of k that are three less than a prime equivalent to 11 mod 12, and we show computer-assisted computations of many values of k for which equality fails.

Title: Chromatic Symmetric Functions and Graphs
Presenter: Sofia Martinez Alberga
Affiliation: University of California, Riverside
Abstract: Properties of chromatic symmetric functions for specific graph classes have long been studied. One of the fundamental questions is whether a chromatic symmetric function uniquely determines a tree. This question was posed first by Stanley in 1995 and it remains an open problem, although it has been answered in the affirmative for a number of special classes of trees including caterpillars and spiders. Here we show the result holds for generalized spiders (i.e. line graphs of spiders) too, thereby extending the work of Martin, Morin and Wagner. A second fundamental question is whether a chromatic symmetric function is e-positive. Here, in this presentation, we establish that certain classes of generalized spiders (i.e. those known as generalized nets) are not e-positive, and we use yet another class of generalized spiders to construct a counterexample to a conjecture involving the e- positivity of claw-free, P4-sparse graphs, showing that Tsujie's result cannot be extended to this set of graphs. Finally, a fourth class of generalized graph, the generalized lollipop, is shown to be e-positive. This generalizes the work of Gebhard and Sagan and Cho and Huh. This research was conducted in the 2018 Fields Undergraduate Summer Research Program, held at the Fields Institute in Toronto, Canada.

Title: On Seifert Graphs and Braid Index
Presenter: Van Pham
Affiliation: University of North Carolina at Charlotte
Abstract: From a link diagram, one can construct its Seifert graph, which gives some information about the original link diagram that helps determining the braid index of the link diagram. However, some details about the original links are lost in the process. We are interested in recovering some of these details in Seifert graphs. Although our method does not work for all links, it provides an upper bound for the braid index.

Title: Research into Homology of Complexes
Presenter: William Pittington
Affiliation: New Mexico State University
Abstract: My motivation for conducting this research is to develop an understanding of the relationship between graphs, networks, and algebraic topology. I have been investigatring various types of graphs including complete graphs, cycles and Petersen graphs. I am interested in finding the neighborhood complex $N(G)$ associated with a graph G whose n-simplices are the set of n vertices that share a common neighbor.I am currently doing calculations with the simplicial chain complex associated with $N(G)$. One goal is to extend my research beyond neighborhood complexes and learn about different types of simplicial complexes. The primary long term goal is to use these tools and concepts to compute the homology of these complexes. Through these computations I will be able to learn more about the classification of graphs and topological spaces.

Title: Random Positroid Statistics
Presenter: Sophie Quynn
Affiliation: University of California, Davis
Abstract: The study of randomly sampled combinatorial objects has a long and rich history that has produced probabilistic models for graphs, simplexes, and matroids, to name a few. In this vein, we aim to explore the behavior of a well-behaved subclass of matroids called positroids. First introduced by Postnikov, positroids come from the arrangement of the totally nonnegative Grassmannian. They are indexed by several combinatorial objects, such as Grassman necklaces and decorated permutations. We randomly sample positroids via their decorated permutation representation, and aim to describe the distribution of various statistics such as rank, basis count, and circuit count. A conjecture for Welsh states that the sequence of rank r matroids over ground set E of size n, where $1 \leq r \leq n$, is unimodel. Our data collected so far supports this conjecture for positroids. We work towards affirmatively proving Welsh's conjecture for all positroids. This is a joint work with Anastasia Chavez and Chris O'Neill.

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