

# USTARS

underrepresented students in topology and algebra research symposium

**Sam Houston State University**

Huntsville, Texas

April 15-17, 2016



# Agenda

## Friday, April 15

### Arrival to USTARS 2016

Location: Sam Houston State University Hotel

## Saturday, April 16

- 8:00-12:00pm **Registration**  
Lee Drain Building Atrium
- 9:00-9:30am **Welcome and Opening Remarks**  
Dr. Candice Price, Sam Houston State University  
Location: Lee Drain Building 214
- 9:30-10:00am **Session I**  
*Minimum rank of graphs with loops*  
Chassidy Bozeman, Iowa State University  
Location: Lee Drain Building 215
- Artin's conjecture for unramified extensions of  $\mathbb{Q}_p$*   
Luis Sordo Vieira, University of Kentucky  
Location: Lee Drain Building 208
- A decomposition of the group algebra of the hyperoctahedral group*  
Drew Tillis, University of North Texas  
Location: Lee Drain Building 220
- 10:15-10:45am **Session II**  
*The topology of liouville foliation for Goryachev-Chaplygin case and the Poincare case*  
Fariba Khoshnasib, University of Texas at Dallas  
Location: Lee Drain Building 215
- Cardinality Restrictions on Noetherian Spectra*  
Cory Colbert, University of Texas at Austin  
Location: Lee Drain Building 208
- Polydegree Properties of Polynomial Automorphisms*  
Kaitlyn Perry, The University of Alabama  
Location: Lee Drain Building 220
- 10:45-11:15am **Coffee Break**  
Location: Lee Drain Building Atrium
- 11:15-12:30pm **Invited Faculty Speaker**  
*My Combinatorial Journey through Algebra and Topology*  
Dr. a@a Mbirika, The University of Wisconsin - Eau Claire  
Location: Lee Drain Building 214

## Saturday, April 16

12:30-1:30pm **Lunch and Mentoring Panel**  
Location: Lee Drain Building 214  
**Panelists:**  
Dr. Lori Alvin, Bradley University  
Nickolas Castro, The University of Georgia  
Dr. Garrett Jones, The University of Wisconsin - Stevens Point  
Dr. Cris Negron, Louisiana State University  
Dr. Rebecca Garcia, Sam Houston State University  
**Moderator:** Dr. Syvillia Averett, College of Coastal Georgia

1:30-2:00pm **Session III**  
*The Line Scheme of  $A(\alpha)$*   
Derek Tomlin, University of Texas at Arlington  
Location: Lee Drain Building 215

*Galois module of the square root of the inverse different over maximal orders*  
Sin Yi Tsang, University of California Santa Barbara  
Location: Lee Drain Building 208

*Free Inverse Semigroupoids and Their Closed Inverse Subsemigroupoids*  
Veny Liu, The University of Alabama  
Location: Lee Drain Building 220

2:15-2:45pm **Session IV**  
*Lifts of Frobenius on Arithmetic Jet Spaces of Schemes*  
Erik Medina, University of New Mexico  
Location: Lee Drain Building 215

*The Space of Biorders on Some Solvable Groups*  
Kelli Karcher, Virginia Tech  
Location: Lee Drain Building 208

*An Application of Symplectic Integration on a Lie Group of an  $N$ -body Problem*  
William Frazier, East Tennessee State University  
Location: Lee Drain Building 220

3:00-3:30pm **Session V**  
*Extending the  $\log(2k - 1)$  Theorem*  
Rosemary Guzman, University of Illinois at Urbana-Champaign  
Location: Lee Drain Building 215

*Latin Squares and Linearity*  
Stefanie Wang, Iowa State University  
Location: Lee Drain Building 208

*Curvature on the Integers*  
Malik Barrett, University of New Mexico  
Location: Lee Drain Building 220

## Saturday, April 16

3:30-3:45pm **Group Photo**

3:45-5:00pm **Distinguished Graduate Speaker**  
*Coxeter groups, peak sets, and root systems*  
Alexander Diaz-Lopez, University of Notre Dame  
Location: Lee Drain Building 214

5:15-7:00pm **Poster Session and Informal Networking**  
Location: Lee Drain Building Atrium

7:00-8:30pm **Symposium Banquet**  
Location: Walker Education Center -President Houston Room

## Sunday, April 17

9:30-10:00am **Session VI**  
*The irreducible modules of Yokonuma-type Hecke algebras*  
Ojas Dave, University of North Texas  
Location: Lee Drain Building 215

*Supercharacters and combinatorics of unipotent matrix groups*  
Megan Ly, University of Colorado Boulder  
Location: Lee Drain Building 208

*Dehn-Sommerville relations and the Catalan matroid*  
Nicole Yamzon, San Francisco State University  
Location: Lee Drain Building 220

*The Image of the Witten Genus*  
John Mosley, University of Kentucky  
Location: Lee Drain Building 212

10:20 - 11:20am **Distinguished Graduate Speaker**  
*Random groups and cubulations*  
Yen Duong, University of Illinois at Chicago  
Location: Lee Drain Building 214

11:30-12:45pm **Lunch and Professional Development Round Table**  
Location: Lee Drain Building 214

12:45 - 1:00pm **Symposium Closing**

## Presentation Abstracts

**Title:** Curvature on the Integers

**Presenter:** Malik Barrett

**Affiliation:** University of New Mexico

**Abstract:** Starting with a symmetric/antisymmetric matrix with integer coefficients (which we view as an analogue of a metric/form on a principal bundle over the “manifold  $\text{Spec } \mathbb{Z}$ ”) I’ll introduce arithmetic analogues of Chern connections and their curvature (in which usual partial derivative operators acting on functions are replaced by Fermat quotient operators acting on integer numbers); curvature is introduced via the method of “analytic continuation between primes”. I’ll also show various results for curvature; morally,  $\text{Spec } \mathbb{Z}$  will appear as “intrinsically curved.” This is part of a theory that can be viewed as the first steps in developing a “differential geometry of  $\text{Spec } \mathbb{Z}$ ”.

**Title:** Minimum rank of graphs with loops

**Presenter:** Chassidy Bozeman

**Affiliation:** Iowa State University

**Abstract:** Loop graph  $G$  is a finite undirected graph that allows loops but does not allow multiple edges. The set  $S(G)$  of real symmetric matrices associated with a loop graph  $G$  of order  $n$  is the set of symmetric matrices  $A = [a_{ij}] \in \mathbb{R}_{n \times n}$  such that  $a_{ij} \neq 0$  if and only if  $ij \in E(G)$ . The minimum rank of a loop graph  $G$  is the minimum of the ranks of the matrices in  $S(G)$ . We characterize loop graphs that have minimum rank equal to the order of the graph.

**Title:** Cardinality Restrictions on Noetherian Spectra

**Presenter:** Cory Colbert

**Affiliation:** University of Texas at Austin

**Abstract:** Recall that if  $R$  is a commutative ring,  $\text{Spec } R$  is the set of all its prime ideals. There is a natural order that may be placed on  $\text{Spec } R$  which turns it into a poset – the order induced by inclusion of prime ideals. If  $R$  is required to be Noetherian, some interesting inclusions are forced to exist in  $\text{Spec } R$ . These inclusion relations have been studied for decades, and many surprising results have been proved. We will discuss some of these inclusions and discuss what happens when one allows the ring to have large cardinality.

**Title:** The irreducible modules of Yokonuma-type Hecke algebras.

**Presenter:** Ojas Dave

**Affiliation:** University of North Texas

**Abstract:** In this talk, I will construct the irreducible modules for a Yokonuma type Hecke algebra. Additionally, I will discuss algebraic and combinatoric tools needed for the construction. These tools include b-partitions of  $n$ , standard Young Tableaux, specializations, and more.

**Title:** Coxeter groups, peak sets, and root systems

**Presenter:** Alexander Diaz-Lopez\*

**Affiliation:** University of Notre Dame

**Abstract:** Coxeter groups can be realized as groups of reflections of certain geometric structures called root systems. The most common examples are dihedral groups, symmetric groups, and Weyl groups of semi-simple complex Lie algebras. In this talk I will present a brief introduction to Coxeter groups and their importance in other fields, for example combinatorics, Lie theory, and algebraic geometry. We will conclude the talk discussing two of my current research projects: peak sets of classical Coxeter groups, and root systems of reflection systems.

**Title:** Random groups and cubulations

**Presenter:** Yen Duong\*

**Affiliation:** University of Illinois at Chicago

**Abstract:** Geometric group theory is a relatively young field of mathematics that combines combinatorics, algebra, geometry, and topology. One major goal of GGT is to find spaces on which groups act “nicely”. This talk will introduce geometric group theory, as well as another young topic, random groups. Our end goal is to find a geometric space on which a random group acts nicely, using a canonical construction.

**Title:** An Application of Symplectic Integration on a Lie Group of an N-body Problem

**Presenter:** William Frazier

**Affiliation:** East Tennessee State University

**Abstract:** Molecular Dynamics (MD) is the numerical simulation of a large system of interacting molecules, and one of the key components of an MD simulation is the numerical estimation of the solutions to a system of nonlinear differential equations. Such systems are very sensitive to discretization and round off error, and correspondingly, standard techniques such as Runge-Kutta methods can lead to poor results. However, MD systems are conservative, which means that we can use Hamiltonian mechanics and symplectic transformations (also known as canonical transformations) in analyzing and approximating solutions. This is standard in MD applications, leading to numerical techniques known as symplectic integrators, and often, these techniques are developed for well-understood Hamiltonian systems such as Hills lunar equation. In this presentation, we explore how well symplectic techniques developed for well-understood systems (specifically, Hills Lunar equation) address discretization errors in MD systems which are fail for one or more reasons.

**Title:** Extending the  $\log(2k - 1)$ -Theorem

**Presenter:** Rosemary Guzman

**Affiliation:** University of Illinois at Urbana-Champaign

**Abstract:** In this talk, I discuss current work that expands the scope of the  $\log(2k - 1)$ -Theorem of Anderson, Canary, Culler and Shalen. This was a seminal result in that it articulated a relationship between a set of  $k$  freely-generating isometries of hyperbolic 3-space and how they interacted with points in hyperbolic 3-space; namely, under certain conditions, at least one of the given isometries must move a point  $P$  by a distance  $\geq \log(2k - 1)$ . The result lay the foundation for future novel geometric-topological results. Here I discuss an expansion of the theorem, wherein we consider sets of length- $n$  words contained in a rank-2 free group  $\Xi$  on 2 letters (one can consider  $\geq 2$  letters via the same methods), and present a generalized version that restricts how these isometries displace points in hyperbolic 3-space. This has application to classifying certain hyperbolic 3-manifolds in that the volume of the resulting manifold  $M$  gotten by quotient of hyperbolic 3-space with  $\Xi$ , is expected to have a bounded volume which is improved from known volume bounds.

**Title:** The Space of Biorders on Some Solvable Groups

**Presenter:** Kelli Karcher

**Affiliation:** Virginia Tech

**Abstract:** A group is said to be biorderable if it has a total order invariant under left and right multiplication. These orders can be given a topology and is called the space of biorders on this group. There has been intensive study on the space of left-orders recently, but less on the space of biorders. We will focus on solvable groups to show under certain conditions the space of biorders is either finite or homeomorphic to the Cantor set.

**Title:** The topology of liouville foliation for Goryachev-Chaplygin case and the Poincare case

**Presenter:** Fariba Khoshnasib

**Affiliation:** University of Texas at Dallas

**Abstract:** Study of rigid body motion is applied in controlling the dynamics of a satellite on a circular earth orbit, a gyroscope or a pendulum. In order to simplify the analysis, we usually work on models of symmetric rigid bodies. One such example which has a completely integrable system is the Goryachev-Chalpygin top.

The topology of Liouville foliation on  $SO(4)$  corresponding to stability of periodic solutions of the Goryachev-

Chaplygin system is studied and illustrated using its bifurcation diagram and the topological concept of a bifurcation complex is discussed. Also, this idea has been applied to Poincaré's system of motion of solid filled with fluid.

The idea of bifurcation complex can be applied to other integrable systems with two degrees of freedom, too. This topological approach can be applied to find non-degenerate solutions of integrable systems.

**Title:** Free Inverse Semigroupoids and Their Closed Inverse Subsemigroupoids

**Presenter:** Veny Liu

**Affiliation:** The University of Alabama

**Abstract:** Inverse semigroupoids are generalizations of inverse semigroups and groupoids. A free objects in the category of inverse semigroupoids are interesting themselves and they form the foundation for studying inverse semigroupoids combinatorially. Using the immersion of graphs from Stallings Foldings, we introduce Stallings kernel. We show that the structure of the closed inverse subsemigroupoids of free inverse semigroupoid is to some extent similar to the structure of subgroups of a free group. In particular, there are analogues of the Nielsen-Schreier theorem and Howson's theorem. In contrast to the situation in a free group, every finitely generated closed inverse subsemigroupoid of a free inverse semigroupoid  $F$  has finite index (whether or not  $F$  is finitely generated).

**Title:** Supercharacters and combinatorics of unipotent matrix groups

**Presenter:** Megan Ly

**Affiliation:** University of Colorado Boulder

**Abstract:** Much of what we can learn from the representation theory of a finite group is found in the character theory. However, the character theory of the finite group of unipotent upper triangular matrices is well-known to be a “wild” problem. In situations like this, where the full character theory is not attainable, it is useful to consider a coarser version of the usual character theory known as a supercharacter theory. This talk explores a supercharacter theory on the finite group of unipotent upper triangular matrices and some of the stunning combinatorics based on set partitions that arises.

**Title:** My Combinatorial Journey through Algebra and Topology

**Presenter:** aᄁa Mbirika\*\*

**Affiliation:** The University of Wisconsin - Eau Claire

**Abstract:** As the talk title suggests, this talk is about my combinatorial journey through the research areas of algebra and topology. The journey is divided into three parts corresponding to three respective areas of my life: before, during, and after graduate school. Though my interests in graduate school have always focused on topics in algebra and topology, combinatorics was the tool that I used most to solve research problems in these two fields. And indeed it was and still is combinatorics that I enjoy the most. This talk is about the circumstances in my life that led me to where I am now—proudly calling myself a combinatorialist. Time-permitting, I will talk about my most recent research projects with undergraduates at the University of Wisconsin-Eau Claire.

**Title:** Lifts of Frobenius on Arithmetic Jet Spaces of Schemes

**Presenter:** Erik Medina

**Affiliation:** University of New Mexico

**Abstract:** Lifts of Frobenius on  $p$ -adic formal schemes may be viewed as arithmetic analogues of vector fields on manifolds. In particular, vector fields on the tangent bundle of a manifold (appearing for instance in Hamiltonian mechanics) have as arithmetic analogues lifts of Frobenius on arithmetic jet spaces  $J^1(X)$  of schemes. My main research result has been to show the nonexistence of lifts of Frobenius on the arithmetic jet spaces of projective space. In contrast, there do exist, however, smooth vector fields on the tangent bundle of the projective line. I have studied these and related questions.

**Title:** The Image of the Witten Genus

**Presenter:** John Mosley

**Affiliation:** University of Kentucky

**Abstract:** The Witten genus is a variant of Ochanines elliptic genus and can be interpreted as the index of the Dirac operator on the free loop space of a manifold. The natural domain of the Witten genus is string cobordism. The image of this ring under the Witten genus is difficult to compute, but is of some interest. A more tractable problem may be to compute the image of Spin- and SU-cobordism under the Witten genus. In this talk, we will discuss recent progress on this question.

**Title:** Polydegree Properties of Polynomial Automorphisms

**Presenter:** Kaitlyn Perry

**Affiliation:** The University of Alabama

**Abstract:** The group of automorphisms of the affine plane has the structure of an amalgamated free product of the triangular and affine subgroups. This leads us to the polydegree: the unique sequence of degrees of the triangular automorphisms in the amalgamated free product decomposition of the automorphism. The automorphism group is also endowed with the structure of an infinite dimensional algebraic variety. The interaction between these two structures is not well understood. We will discuss the general problem and a new result that a class of automorphisms with a polydegree of length one are contained in the closure (in the Zariski topology) of a set of automorphisms with a polydegree of length 2.

**Title:** Artin's conjecture for unramified extensions of  $\mathbb{Q}_p$ .

**Presenter:** Luis Sordo Vieira

**Affiliation:** University of Kentucky

**Abstract:** We provide a proof of Artin's conjecture for diagonal forms over degree  $> 1$  unramified extensions of  $\mathbb{Q}_p$  with  $p > 2$ . Namely, given a diagonal form of degree  $d$  in more than  $d^2$  variables, there exists a zero in the  $p$ -adic field. The case for  $\mathbb{Q}_p$  was proved by Davenport and Lewis, and larger degree unramified cases by collaborative work in progress with David B. Leep.

**Title:** A decomposition of the group algebra of the hyperoctahedral group

**Presenter:** Drew Tillis

**Affiliation:** University of North Texas

**Abstract:** A complete set of orthogonal idempotents of the group algebra of a finite group leads to a decomposition of the group algebra as a sum of submodules. For a hyperoctahedral group  $W$ , such a decomposition of the group algebra of  $W$  arises from a complete set of idempotents in a particular subalgebra of the group algebra. In 2007, Cedric Bonnafé asked if the submodules in this decomposition could be expressed as induced modules. In this talk, I will discuss a special case and show that the ideal in question is indeed an induced module. It is expected that a positive answer in this special case will lead to a positive answer to Bonnafé's question.

**Title:** The Line Scheme of  $A(\alpha)$

**Presenter:** Derek Tomlin

**Affiliation:** University of Texas at Arlington

**Abstract:** The classification of regular algebras of global dimension four (so-called quantum  $\mathbb{P}^3_s$ ) has been a source of motivation for modern research in noncommutative algebra. In this talk, we discuss and describe the line scheme of a family of algebras whose generic member is a candidate for being a generic quadratic quantum  $\mathbb{P}^3$ .

**Title:** Galois module of the square root of the inverse different over maximal orders

**Presenter:** Sin Yi (Cindy) Tsang

**Affiliation:** University of California, Santa Barbara

**Abstract:** Let  $K$  be a number field with ring of integers  $\mathcal{O}_K$  and let  $L/K$  be a finite Galois extension with group  $G$ . Any fractional  $\mathcal{O}_K$ -ideal in  $L$  may be viewed as an  $\mathcal{O}_K G$ -module. It is a classical problem to study

the  $\mathcal{O}_K G$ -structure of the ring of integers of  $L$ , and recently people have also looked at the ideal whose square is the inverse different  $A_{L/K}$  of  $L/K$ . However, it is in general difficult to work over  $\mathcal{O}_K G$ . One way around this is to extend scalars to a maximal  $\mathcal{O}_K$ -order  $\mathcal{M}(KG)$  in  $KG$ . I will talk about a new result that I have proved concerning the  $\mathcal{M}(KG)$ -structure of  $A_{L/K}$  when  $G$  is abelian.

**Title:** Latin Squares and Linearity

**Presenter:** Stefanie Wang

**Affiliation:** Iowa State University

**Abstract:** Familiar example of a Latin square is the Cayley table for a finite group or a completed Sudoku puzzle. Each element appears only once in each row and column. A quasigroup  $(Q, *)$  is a set with a binary operation  $*$  such that for any two elements  $a, b$  in  $Q$ , there exist unique  $x, y$  in  $Q$  that satisfy  $a*x=b$  and  $y*a=b$ . The multiplication table for a finite quasigroup is a Latin square. A notable feature of quasigroups is that they are not always associative. The integers under subtraction form a non-associative quasigroup.

The number of finite quasigroups is so large that classifying them is a difficult task. There are 1,130,531 distinct quasigroups with 6 elements. We narrow our scope to quasigroups of a linear type, namely piques. These are quasigroups with a pointed idempotent  $e$  that satisfies  $e * e = e$ .

The purpose of this talk is isomorphism classes of piques. We bring in methods of classification from group representation theory and character theory for a starting point. From there, we focus on what data is lost in translation when we build complex representations from abelian group representations so that we can build up a comprehensive method of identifying isomorphism classes of piques.

**Title:** Dehn–Sommerville relations and the Catalan matroid

**Presenter:** Nicole Yamzon

**Affiliation:** San Francisco State University

**Abstract:** The  $f$ -vector of a  $d$ -dimensional polytope  $P$  stores the number of faces of each dimension. When  $P$  is simplicial the Dehn–Sommerville relations condense the  $f$ -vector into the  $g$ -vector, which has length  $\lceil \frac{d+1}{2} \rceil$ . Thus, to determine the  $f$ -vector of  $P$ , we only need to know approximately half of its entries. This raises the question: Which  $(\lceil \frac{d+1}{2} \rceil)$ -subsets of the  $f$ -vector of a general simplicial polytope are sufficient to determine the whole  $f$ -vector? We prove that the answer is given by the bases of the Catalan matroid.

\*\* Invited Faculty Speaker

\* Distinguished Graduate Student

## *Poster Abstracts*

**Title:** Girth for Finitely Generated Groups

**Presenter:** Faraad Armwood

**Affiliation:** North Dakota State

**Abstract:** The notion of girth of a finitely generated group was first introduced by S.Schleimer. Many authors have studied this notion since then; it has become a useful tool in studying dynamical properties of group actions, computing free limits of groups, understanding graph theoretic structures of finite groups, etc.

In this work we present a simple argument which provides upper bounds for the girth of finite groups. Then we restrict our attention to some examples of finite simple groups and produce finer upper bounds (or exact values).

We also introduce the notion of Shreier girth and study its properties. The Shreier girth can be strictly less than the girth. In particular, there exist groups of infinite girth which have a finite Shreier girth. We prove for some interesting classes of groups that their Shreier girth is infinite

**Title:** Pseudospectra and Matrix Behavior

**Presenter:** Nicholas Camacho

**Affiliation:** Florida Gulf Coast University

**Abstract:** Whereas the spectrum of a normal matrix determines its behavior, the pseudospectrum offers an alternative to better understand the behavior of matrices that are nonnormal. In this study, we investigate the relationship between pseudospectra and matrix behavior. In particular, we investigate the implications of matrices with equal spectral norm pseudospectra.

**Title:** Lattice Point Visibility on Generalized Lines of Sights

**Presenter:** Sara DeBrabander and Michelle Gebert

**Affiliation:** The University of Wisconsin - Eau Claire

**Abstract:** Integer lattice point visibility has been studied since 1971 on straight lines through the origin with rational slopes. Harris, Kubik, and Mbirika generalize this notion of lines of sights to include all curves through the origin given by functions of the form  $f(x) = \frac{a}{b}x^n$  where  $\frac{a}{b} \in \mathbb{Q}$  and  $n \in \mathbb{Z}^+$ . Many questions remain open in this new setting of generalized lines of sights. In the classic setting where  $n = 1$  a lattice point is visible if there exists no other lattice point between the origin and the point. The visible points are the points  $(x, y)$  where  $\gcd(x, y) = 1$ . We explore the process of identifying the first visible lattice point on a given curve  $f(x) = \frac{a}{b}x^n$  in this new generalized setting. Also we explore the form of any given lattice point on  $f$  in this setting. This allows us to find the number of lattice points on  $f$  between the origin and a given lattice point  $(x, y)$ . We compare these findings to the classic setting.

**Title:** Modeling a Stochastic Shortest Path Problem

**Presenter:** Lucas Everham

**Affiliation:** Florida Gulf Coast University

**Abstract:** The goal of this research is to better understand the approach to and algorithms required for modeling stochastic shortest path problems, both with and without recourse, and to build such a model using Python programs. It begins with a discussion of some of the fundamental concepts necessary for approaching problems of this nature and further looks into past research on problems such as these (e.g., the Canadian Traveller Problem). The specific scenario modeled herein is one in which the goal is to determine shortest paths via bicycle considering wind velocity and distance.

**Title:** An Exploration of Lie Algebras and Kostant's Weight Multiplicity Formula

**Presenter:** Brett Harder

**Affiliation:** Moravian College

**Abstract:** A Lie algebra  $L$  is a vector space over a field with a bilinear product called the Lie bracket;  $[x, y] = xy - yx$  for all  $x, y \in L$ . The special linear Lie algebra of order  $n$ ,  $\mathfrak{sl}(n, \mathbb{C})$ , is the Lie algebra of  $n \times n$  matrices with vanishing trace. This project aims to explore the properties of Lie algebras of this type and calculate weight multiplicities through the use of Kostant's Weight Multiplicity Formula:

$$m(\lambda, \mu) = \sum_{\sigma \in W} \epsilon(\sigma) \wp(\sigma(\lambda + \rho) - (\mu - \rho)) \quad (1)$$

**Title:** Methods for Upper Bounds on the Birank Number of  $(3 \times n)$  Grid Graphs

**Presenter:** Shane Harder

**Affiliation:** Moravian College

**Abstract:** Given a graph  $G$ , a function  $f : V(G) \rightarrow \{1, 2, \dots, k\}$  is a  $k$ -biranking if  $f(u) = f(v)$  implies that every  $u$ - $v$  path contains vertices  $x$  and  $y$  such that  $f(x) < f(u) < f(y)$ . The birank number is the minimum  $k$  such that  $G$  has a  $k$ -biranking. The birank number for path and ladder graphs are known. However the birank number for  $(3 \times n)$  grid graphs are not. We have determined the birank number for small values of  $n$  and created methods to closely approximate the birank number for larger values of  $n$  by upper bounds.

**Title:** Broadcast and Domination in Radial Graphs

**Presenter:** Farhana Huda

**Affiliation:** Florida Gulf Coast University

**Abstract:** Let  $G = (V, E)$  be a graph with vertex set  $V$  and edge set  $E$ . A *broadcast* on a graph  $G$  is a function  $f : V \rightarrow \{0, \dots, \text{diam}(G)\}$  such that for every vertex  $v \in V(G)$ ,  $f(v) \leq e(v)$ , where  $\text{diam}(G)$  denotes the diameter of  $G$  and  $e(v)$  denotes the eccentricity of vertex  $v$ . A *dominating broadcast* is a broadcast for which every vertex in  $G$  hears at least one broadcasting center. The *cost* of a broadcast is the value  $f(V) = \sum_{v \in V} f(v)$ , and the *broadcast domination number*  $\gamma_b(G)$  is the minimum cost of any dominating broadcast. Graphs for which  $\gamma_b(G) = \text{rad}(G)$  are called *radial graphs* where  $\text{rad}(G)$  is the *radius* of the graph. A *radial tree* is a radial graph that is a tree. In this project, we will use the characterization of radial trees by Herke and Mynhardt to find classes of radial graphs. We proved our lemma for the cycles to be radial or not. We applied our algorithm on graphs to have spanning trees which have  $\gamma_b(T^*) \leq \gamma_b(T)$  to classify  $G$  is radial or not. We show some examples to support our conjectures and illustrate how our results can be applied.

**Title:** Techniques toward a categorification of  $\mathfrak{sl}_2$ -Verma modules

**Presenter:** Mee Seong Im

**Affiliation:** United States Military Academy

**Abstract:** Categorification is a method to construct deeper and intricate structure associated to simple objects, such as the natural numbers or the deformation of the universal enveloping algebra of a Kac-Moody algebra, such that decategorification returns to us the original objects, with the additional structure removed. I will talk about certain known techniques using modules, which are then used to categorify Verma modules for  $\mathfrak{sl}_2$ . This is joint with Ben Cox.

**Title:** On Invariants and Semi-Invariants of Filtered Quiver Representations of  $n$ -Kronecker and  $n$ -Jordan Quivers

**Presenter:** Lisa Jones

**Affiliation:** United States Military Academy

**Abstract:** Given the standard filtration of complex vector spaces, we define a filtered quiver representation as a quiver representation with linear maps that preserve the filtration of the vector spaces. We describe the invariant and semi-invariant polynomials for filtered quiver spaces of  $n$ -Kronecker and  $n$ -Jordan quivers. This problem has interesting implications for algebraic coding theory, particularly for the generalization of a theorem of MacWilliams on the weight enumerators for a code and its dual.

**Title:** Combinatorial Navier-Stokes Equation in 3-dimension

**Presenter:** Aradhana Kumari

**Affiliation:** The Graduate Center, CUNY

**Abstract:** We intend to study the Navier-Stokes equation in 3-dimensions qualitatively and quantitatively. Using tools from algebraic topology one can construct a combinatorial setting to approximate the continuum. One usually discretizes time by using finite steps and spatial parameter by using finite spatial grids. Differential forms, the hodge star operator and exterior derivative  $d$  operator which are needed to write the Navier-Stokes equation must now be replaced by suitable analogues from topology. There are many choices involved in this discretization procedure. My work first attempts to formulate the conditions that imply a nonlinear analogue of Lax-Richtmyer equivalence between stability of numerical computation and its convergence to a genuine solution. Secondly it tries to apply these to optimal choices of discretization. A further ingredient is the tool of infinity structures from algebraic topology which provides consistency to discretization at different scales. This is a new ingredient not yet used in numerical studies. The hope is that this infinity structure might give some insight as for example discrete heat equation gives a new insight called random walks.

**Title:** On Broadcast Domination Numbers of Grids

**Presenter:** Karen Marino

**Affiliation:** Florida Gulf Coast University

**Abstract:** In 2006 Dunbar, Erwin, Haynes, Hedetniemi, and Hedetniemi defined a *broadcast* on a graph as a function  $f : V \rightarrow \{0, 1, \dots, \text{diam}(G)\}$  such that for every vertex  $v \in V, f(v) \leq e(v)$ , where  $\text{diam}(G)$  denotes the diameter of  $G$  and  $e(v)$  denotes the eccentricity of  $v$ . They defined the *cost* of a broadcast as the value  $f(V) = \sum_{v \in V} f(v)$ . The upper and lower broadcast domination numbers are the maximum (and minimum) cost of all minimal dominating broadcasts. In this paper, we explore the upper and lower broadcast domination number of graphs. We find the exact formulas for the upper broadcast domination numbers of grid graphs and cycles. We also identify a family of graphs for which the distance between lower and upper broadcast domination numbers is maximized.

**Title:** I Want to be the Very Best: Spectral Analysis in Pokmon Choice

**Presenter:** Paul Myrin

**Affiliation:** Florida Gulf Coast University

**Abstract:** In the world of competitive Pokmon, competitors create teams by choosing six unique Pokmon out of over seven hundred possible Pokmon. When observing the Pokmon chosen of various teams, it is apparent that certain Pokmon are chosen more often than other Pokmon. Further observation reveals that, as with 1 Pokmon, certain pairs of 2 Pokmon are chosen more than other pairs of 2 Pokmon. This pattern can be observed all the way up to noticing certain entire teams are chosen more often than other entire teams. These observations lead to the following question: do competitors find it more important to have 1 specific Pokmon on a team, 2 specific Pokmon on a team, an entire specific team of Pokmon, or anything in between? This question can be solved through an application of linear algebra known as spectral analysis.

**Title:** Leamer Monoids and the Huneke-Wiegand Conjecture

**Presenter:** Miguel Landeros and Karina Pena

**Affiliation:** California State Polytechnic University, Pomona

**Abstract:** The Huneke-Wiegand Conjecture is a long standing open problem in commutative algebra from the early 90's regarding torsion submodules of tensor products. The recent work of P. Garcia-Sanchez and M. Leamer provides an approach towards a partial solution of the Huneke-Wiegand Conjecture by reducing certain cases to the existence of particular atoms in monoids. In their work, Garcia-Sanchez and Leamer construct a new class of monoids, known as Leamer monoids, whose elements correspond to arithmetic sequences contained in a numerical monoid. Their reduction shows that special cases of the Huneke-Wiegand Conjecture are equivalent to the existence of an irreducible element of the form  $(x, 2)$ . In this poster, we focus on identifying these atoms in Leamer monoids whose underlying numerical monoid is symmetric and generated by either an arithmetic or generalized arithmetic sequence. We highlight a visual technique developed to identify the location of the irreducible elements which led to an existence proof of an irreducible element of the form  $(x, 2)$  for any Leamer monoid generated by a symmetric numerical monoid whose generators form a generalized arithmetic sequence.

We end with a discussion of challenges and conjectures regarding Leamer monoids generated by either a non-symmetric numerical monoids or ones with arbitrary generators.

**Title:** A Combinatorial Problem on Finite Abelian Groups

**Presenter:** Darleen Perez-Lavin

**Affiliation:** University of Kentucky

**Abstract:** In 1968, John E. Olsen presented a combinatorial problem on finite abelian groups. He provided results of the davenport constant for specific finite abelian groups with an interesting application in linear algebra.

**Title:** Knot and Link Tricolorability

**Presenter:** Molly Petersen

**Affiliation:** The University of Wisconsin-Eau Claire

**Abstract:** Our team has been researching the colorability of knots, an invariant used in classification. We focused on the  $n$ -Whitehead double and Pure double of knots that have been observed in circular DNA. Representing the knot as a matrix allows us to determine its colorability, and we are interested in knots that are tricolorable. We have found a pattern in the colorabilities of Whitehead doubles of certain knots when twists are added within the link. We have developed conjectures about these, and have proven tricolorability for the Whitehead double of knot  $5_1$ . We are investigating the relationships of different doubling operators and tricolorability.

**Title:** Bivariate Order Polynomials

**Presenter:** Sandra Zuniga-Ruiz

**Affiliation:** San Francisco State University

**Abstract:** In the early 1900s, in an attempt to prove the 4-coloring theorem Birkhoff discovered the number of ways to color a graph, this is known as the chromatic polynomial. In 1970, Richard Stanley introduced a decomposition of the chromatic polynomial using order polynomials. We extend these results using bivariate polynomials. In 2003, Klaus Dohmen, Andre Pnitz, and Peter Tittman introduced a two variable generalization of the chromatic polynomial. Using the generalized bivariate chromatic polynomial we introduce a decomposition using bivariate order polynomials.

*Special thanks to the following USTARS sponsors, partners, and supporters:*

The National Science Foundation

Sam Houston State University

Sam Houston State University Department of Mathematics and Statistics

Florida Gulf Coast University

The Florida Gulf Coast University Mathematics Department

The USTARS Advisory Board

Dr. Dennis Davenport

Dr. Rebecca Garcia

Dr. Kimberly Kendricks

Dr. Brian Loft

Dr. Ulrica Wilson

Kathy McElroy

Angie Burgess