USTARS

underrepresented students in topology and algebra research symposium

Reed College
Portland, Oregon
April 6 - April 8, 2018

REED COLLEGE

Agenda

Friday, April 6, 2018

6:00am -10:00pm	**Arrival to USTARS** Location: AC Hotel by Marriott Portland Downtown

Saturday, April 7, 2018

8:00am -12:00pm	**Registration** Location: Vollum Foyer
9:30am -9:45am	**Welcome and Opening Remarks** Dr. Candice Price, University of San Diego Location: Vollum Lecture Hall
10:00am - 10:30am	**Session I** *A generalization of the free Lie algebra and its corresponding representations* Sarah Brauner, University of Minnesota Location: Biology 019 *Tying Knots to Infinity-categories* Anna Cepek, Montana State University Location: Physics 123 *Differential Forms* Khoi Vo, California State University, Long Beach Location: Psychology 105
10:40 -11:10am	**Session II** *Amenable basis and Simple Basis of infinite dimensional algebra* Rebin Muhammad, Ohio University Location: Biology 019 *w-Knots and the Infinitesimal Alexander Module* Sherilyn Tamagawa, University of California, Santa Barbara Location: Physics 123 *Action dimensions of some simple complexes of groups* Dr. Giang Le, Oregon State University Location: Psychology 105

Saturday, April 7, 2018

11:20-11:50am	**Session III** *The Topology of Liouville Foliations for the Poincare system on the Lie Algebra E(3)* Fariba Khoshnasib, University of Wisconsin - Eau Claire Location: Biology 019 *The Glasner-Pestov problem and topological weakly mixing* Javier Ronquillo Rivera, Ohio University Location: Physics 123 *An Algebraic Definition of the Bridge Number of a Knot* Paul Villanueva, Iowa State University Location: Psychology 105
12:00-2:00pm	**Lunch and Mentoring Panel** Location: Kaul Auditorium **Panelists:** Nelson Colon, Industry Representative Dr. Christina Eubanks Turner, Loyola Marymount University Javier Ronquillo Rivera, Ohio University Dr. Luis Sordo Vieira, UConn Health, Center for Quantitative Medicine **Moderator:** Dr. Syvillia Averett, College of Coastal Georgia
2:00-3:30pm	**Invited Faculty Speaker** *Some applications of topology to data science* Dr. Jose Perea, Michigan State University Location: Vollum Lecture Hall
3:45-5:00pm	**Poster Session and Informal Networking** Location: Kaul Auditorium
5:10-5:30pm	**Group Photo** Location: Vollum Lecture Hall
5:40-6:40pm	**Distinguished Graduate Speaker** *On the signed Euler characteristic property for subvarieties of abelian varieties.* Eva Elduque, University of Wisconsin-Madison Location: Vollum Lecture Hall
7:00-8:30pm	**Symposium Banquet** Location: Kaul Auditorium

Sunday, April 8, 2018

9:30-10:00am	**Session IV** *Two Examples of Integral Extensions* Nicholas Camacho, The University of Iowa Location: Biology 019 *An isomorphism of* $\mathfrak{so}_{2n}[t]$*-modules* Ryan Moruzzi, Jr., University of California, Riverside Location: Physics 123 *Donfigureations on Graphs* Dr. Safia Chettih, Reed College Location: Psychology 105
10:10am-10:40am	**Session V** *Augmented Hilbert series of numerical semigroups* Dr. Christopher O'Neill, University of California, Davis Location: Biology 019 *Dessins d'Enfants and Counting Quasiplatonic Cyclic Surfaces* Charles Camacho, Oregon State University Location: Physics 123 *Lagrangian formalism for equations of mathematical physics and other dierential equations: New Lagrangians, Lie groups and commutative semigroup and monoid* Niyousha Davachi, University of Texas at Arlington Location: Psychology 105
10:50 - 11:50am	**Distinguished Graduate Speaker** *Embedding Problems over* $\mathbb{Z}_p$ *with Kernel* $\mathbb{Z}_{p^2}^n$ Clifford Bridges, University of Colorado Boulder Location: Vollum Lecture Hall
12:00-12:45pm	**Lunch and Symposium Closing** Location: Vollum Lounge

USTARS 2018 Invited Faculty Speaker

Dr. Jose Perea, Michigan State University

USTARS 2018 Invited Faculty Speaker

Jose Perea is an active researcher in the field of computational topology and topological data analysis. Broadly speaking, his work entails applications and adaptations of ideas from algebraic and geometric topology to the study of high-dimensional and complex data. Perea received his B.S. in mathematics from Universidad del Valle in 2006 (Summa cum laude and Valedictorian), a Ph.D. in mathematics from Stanford University in 2011, and held a postdoctoral position as a visiting assistant professor in the department of mathematics at Duke University (2011 to 2015). In spring of 2014, he was a member of the Institute for Mathematics and Applications at the University of Minnesota, during the annual thematic program on scientific and engineering applications of algebraic topology. He has been at Michigan State University since 2015 as an assistant professor with appointments in the Department of Computational Mathematics, Science and Engineering (CMSE) and the Department of Mathematics.

USTARS 2018 Distinguished Graduate Students

USTARS 2018 Distinguished Graduate Student in Algebra

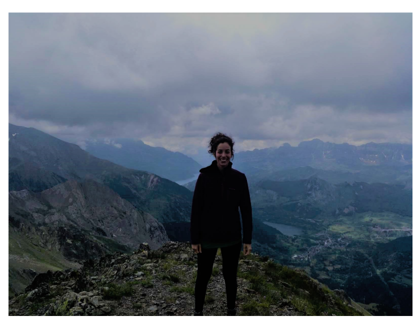
Clifford Bridges, University of Colorado Boulder

Cliff was raised in North Carolina and attended the University of Maryland, Baltimore County (which recently became the only 16 seed to ever beat a 1 seed in the NCAA Men's Basketball tournament), before moving to Colorado to attend the University of Colorado Boulder. He fell into loving math after being annoyed by elementary school teachers who would critique his writing skills but not offer to help improve them. Cliff is currently spending almost all day every day writing his dissertation, so the joke is either on them or on him, he hasn't figured that part out yet. In either case, he grew up with the understanding that some people don't get the same opportunities as others and has worked to even out the playing field, particularly with gender and racial equity issues. Fortunately his love of math has given him at least one tool to help in that battle; teaching and communicating mathematical ideas to others.

Loosely speaking, the inverse Galois problem was the start of Cliff's research career and has led to related interests in algebra, algebraic topology and cohomology. Beyond these more theoretical interests, he would like to learn more about geometry and math education research. More broadly still, Cliff also likes athletics, learning languages, sewing and fashion design, dancing, and sincerity. Please, when you see him, feel free to ask him about any of these topics.

USTARS 2018 Distinguished Graduate Student in Topology

Eva Elduque, University of Wisconsin - Madison



Eva Elduque is a fourth year graduate student at the University of Wisconsin-Madison, under the supervision of Laurentiu Maxim. Eva's interests lie in Topology and Singularities. She did my undergraduate studies at the University of Zaragoza, in her hometown in Spain, and earned her Master's degree from the Autonomous University of Madrid.

Apart from doing math, Eva also enjoys biking, playing basketball, crafting, playing music and hiking in the Pyrenees whenever possible.

Presentation Abstracts

Title: A generalization of the free Lie algebra and its corresponding representations
Presenter: Sarah Brauner
Affiliation: University of Minnesota
Abstract: The free Lie algebra has broad and far-reaching applications in algebraic combinatorics. In this talk, I will discuss a recent generalization of the free Lie algebra using an n-ary commutator introduced by Tamar Friedmann. In particular, I will describe the representations induced by the action of the symmetric group on the multilinear component of this generalized free Lie algebra, which provides a natural analog to the well-studied representations Lie(k). I will then present some recent results and conjectures on the topic by Friedmann, Hanlon, Stanley and Wachs, as well as my contributions to the project.

Title: Embedding Problems over $\mathbb{Z}_p$ with Kernel $\mathbb{Z}_{p^2}^n$
Presenter: Clifford Bridges*
Affiliation: University of Colorado Boulder
Abstract: The embedding problem asks: given groups M and G, as well as a Galois extension K/F with Galois group isomorphic to G, does there exist a Galois extension L over F containing K such that the natural surjection $\mathrm{Gal}(L/F) \to \mathrm{Gal}(K/F)$ has kernel isomorphic to M. This talk will focus on solutions to the embedding problem when G is a cyclic group of order p and M is a finite product of cyclic groups of order p^2. Understanding group extensions is critical in solving embedding problems. If G is generated by σ, then a group extension of M by G is determined by the action by conjugation of σ on M and the value of σ^p, which is an element of M. The action by conjugation of σ on M can be described by a matrix S with entries in $\mathbb{Z}_{p^2}$. This talk will describe conditions which classify affirmative solutions to the embedding problem when S has the form $S = I + pA$ for some A.

Title: Dessins d'Enfants and Counting Quasiplatonic Cyclic Surfaces
Presenter: Charles Camacho
Affiliation: Oregon State University
Abstract: A *quasiplatonic* group is a finite group G which acts topologically on a surface X whose orbit space X/G is homeomorphic to a sphere. We will derive formulas enumerating the distinct topological actions of the cyclic group $G = C_n$ on compact Riemann surfaces X having genus at least two. We find relationships to map enumeration, with specific interest in the Riemann surface structure underlying the embedded map. These maps are bipartite two-cellular embeddings called *dessins d'enfants* (French for children's drawing) or, equivalently, hypermaps. In particular, we aim to count all compact Riemann surfaces, up to conformal equivalence, which admit a regular dessin d'enfant with a cyclic automorphism group. These highly symmetric surfaces are called *quasiplatonic surfaces*, and it is unknown how many quasiplatonic surfaces there are in each genus. On the other hand, the total number of topological actions $QC(n)$ of C_n can be obtained by applying formulas of R. Benim and A. Wootton. We relate $QC(n)$ to the total number $R(C_n)$ of regular dessins with a cyclic group of automorphisms on higher genus surfaces, using results by G. Jones. We also explore the connection between counting topological actions of a quasiplatonic group and enumerating quasiplatonic surfaces up to conformal equivalence.

Title: Two Examples of Integral Extensions
Presenter: Nicholas Camacho
Affiliation: The University of Iowa
Abstract: This talk will be selfcontained and completely accessible to anyone with basic knowledge in abstract algebra. We will introduce a well-known concept in commutative ring theory integral extensions and discuss two examples. Well start by defining integral elements over a ring, and then classify the ring of integers in a cyclotomic field, which has applications in algebraic number theory. We will then discuss when a coordinate ring of a curve is integrally closed, which has applications in algebraic geometry.

Title: Tying Knots to Infinity-categories
Presenter: Anna Cepek
Affiliation: Montana State University
Abstract: Through motivating the definitions of a stratified space, the Ran space, and exit-path infinity-categories within the framework of knots, a connection between knots and the exit-path infinity-category of the Ran space will be illustrated.

Title: Configurations on Graphs
Presenter: Dr. Safia Chettih
Affiliation: Reed College
Abstract: Given a graph Γ, we can construct discretized models for its n-point configuration space that are cubical complexes. In recent work with Daniel Lütgehetmann, we have considered a Światkowski-style discretized model for configurations with sinks, which allow more than one point to occupy certain vertices in the graph. In my talk, I will discuss the construction and some of its implications for the homology of ordered configuration spaces of graphs.

Title: Lagrangian formalism for equations of mathematical physics and other dierential equations: New Lagrangians, Lie groups and commutative semigroup and monoid
Presenter: Niyousha Davachi
Affiliation: University of Texas at Arlington
Abstract:A new class of non-standard Lagrangians is discovered, and it is shown that the members of this class require auxiliary conditions, which is a novel phenomenon in the calculus of variations. The developed Lagrange formalism is compared to the Lie group approach and it is demonstrated that the former is established for all considered ODEs, however, the latter has only limited applications. Moreover, it is also shown that the set of all considered ODEs may be converted into a commutative semigroup or monoid.

Title: On the signed Euler characteristic property for subvarieties of abelian varieties.
Presenter: Eva Elduque*
Affiliation: University of Wisconsin - Madison
Abstract: Franecki and Kapranov proved that the Euler characteristic of a perverse sheaf on a semi-abelian variety is non-negative. This result has several purely topological consequences regarding the sign of the (topological and intersection homology) Euler characteristic of a subvariety of an abelian variety, and it is natural to attempt to justify them by more elementary methods. In this talk, we'll explore the geometric tools used recently in the proof of the signed Euler characteristic property. Joint work with Christian Geske and Laurentiu Maxim.

Title: The Topology of Liouville Foliations for the Poincare system on the Lie Algebra E(3)
Presenter: Fariba Khoshnasib
Affiliation: University of Wisconsin - Eau Claire
Abstract: This is a research built upon special, integrable cases of rigid body motion. The differential equations of motion of such systems are Integrable Hamiltonian ones. The Poincare case is the focus of our interest. This system is completely integrable and the common level set of the first integrals H and F is a smooth manifold that is invariant under the phase flow of the system. Also, we know every connected component of the common level set (symplectic leave) is diffeomorphic to a two-dimensional torus. The bifurcation diagram of such completely integrable system is the region of possible motion on the plane of first integrals together with the image of the critical set. Poincares system models motion of a rigid body filled with viscous fluid and bifurcations of Liouville tori are constructed for this case. This model is a good approximation of motion of the earth.

Title: Action dimensions of some simple complexes of groups
Presenter: Dr. Giang Le
Affiliation: Oregon State University
Abstract: We compute the action dimension of the direct limit of a simple complex of groups for several classes of examples including: 1) Artin groups, 2) graph products of groups, and 3) fundamental groups of aspherical complements of arrangements of affine hyperplanes.

Title: An isomorphism of $\mathfrak{so}_{2n}[t]$-modules
Presenter: Ryan Moruzzi, Jr.
Affiliation: University of California, Riverside
Abstract: In 2010, Hernandez and Leclerc identified a family of prime representations of the quantum affine algebra associated to $\mathfrak{sl}_{n+1}(\mathbb{C})$. In 2015, Chari, Moura, Brito studied the classical limit of that family of prime representations and proved such representations specialize to stable prime Demazure modules for $\mathfrak{sl}_{n+1}[t]$.

Currently, I am working on proving similar results for the Lie algebra $\mathfrak{so}_{2n}(\mathbb{C})$. In this talk, I will introduce one step in the proof which specifically is an isomorphism between particular $\mathfrak{so}_{2n}[t]$ modules, one of which is known as a $V(\xi)$ module, first defined by Chari and Venkatesh in 2013.

Title: Amenable basis and Simple Basis of infinite dimensional algebra
Presenter: Rebin Muhammad
Affiliation: Ohio University
Abstract: The talk will begin by reviewing topics introduced in a 2017 JLMA paper by L.M. Al-Essa, Sergio Lo pez-Permouth and N. M. Muthana. Let A be an infinite-dimensional K-algebra, where K is a field and let B be a basis for A. A basis B is called an amenable basis when K^B become an Amodule in a natural way. Basic modules induced by bases B and C are isomorphic if B and C are mutually congenial. B is congenial to C if the coordinate vectors of the elements of Brepresented with respect to C are summable. If B is congenial to C but C is not congenial to B, one says that B is properly congenial to C. When B is congenial to C and C is congenial to B, we say the bases are mutually congenial. When neither B is congenial to C nor C is congenial to B, the bases are said to be discordant. Basic setting example in previous study of amiability and congeniality was related to polynomial algebras of a single variable. We investigate other algebras such as $k[x, y]$,Kite Algebra and graph algebra . We found that if the algebra is non-commutative then there may exist a basis that is left amenable but not right amenable and vice versa. We give an example in the Kite Algebra for each of these cases. We also discuss the simplicity of basis in polynomial algebras of n variables and show that there is always a simple basis by using results we get from tensor product of algebras and simplicity of basis in polynomial algebra of single variables.

Title: Augmented Hilbert series of numerical semigroups
Presenter: Dr. Chris O'Neill
Affiliation: University of California, Davis
Abstract: A numerical semigroup S is a subset of the natural numbers that is closed under addition, and a factorization of $n \in S$ is an expression of n as a sum of generators of S. The Hilbert series of S is the rational generating function $\sum_{n \in S} t^n$, and there are several characterizations of the numerator in terms of key properties of S. In this talk, we characterize the numerator of several "augmented" Hilbert series, where the coefficient of each t^n is some arithmetic quantity derived from the factorizations of n, such as the maximum factorization length of n or number of distinct factorization lengths of n. The results presented here are from an undergraduate research project from the 2017 SDSU REU.

Title: Some applications of toplogy to data science
Presenter: Dr. Jose Perea
Affiliation: Michigan State University
Abstract: Topology is the branch of mathematics dealing with the description, quantification and classification of shapes from abstract spaces. In the last decade or so, it has been observed that several questions in data science can be answered through the same approach. I'll describe some of these ideas, and will highlight their application in areas such as biology, genetics and computer vision.

Title: The Glasner-Pestov problem and topological weakly mixing
Presenter: Javier Ronquillo Rivera
Affiliation: Ohio University
Abstract: When a class of abelian topological groups has 'enough' non-trivial characters we can study this class of topological groups through a duality theory. Nonetheless for some classes of abelian topological groups the question about having 'enough' non-trivial characters remains open. One of these open problems is due to Glasner and Pestov and states the following: [Glasner-Pestov problem] Let X be compact, $G \subset \mathrm{Homeo}(X)$ an abelian group, such that X has no G-fixed points. Does G admit non-trivial characters?

When G is the closure of the group generated by a single map $T/in\mathrm{Homeo}(X)$ (in the uniform topology $C(X,X)$), one approach to this problem is through studying if the dynamical system (X,T) is topologically weakly mixing. We say that (X,T) is topologically weakly mixing when the product system $(X \times X, T \times T)$ is topologically transitive. The connection between weakly mixing and the Glasner-Pestov problem follows from a theorem proved in a paper by Keynes and Robertson which states that if (X,T) is a minimal system then, (X,T) is topologically weakly mixing if and only if it has no non-trivial equicontinuous factor. This connection is due to the link between non-trivial equicontinuous factors and having non-trivial characters. In this talk we will discuss the Glasner-Pestov problem and describe the above mentioned approach to the problem using the property of topologically weakly mixing.

Title: w-Knots and the Infinitesimal Alexander Module
Presenter: Sherilyn Tamagawa
Affiliation: University of California, Santa Barbara
Abstract: First introduced in the 1920s, the Alexander polynomial is a well-studied knot invariant. The Infinitesimal Alexander Module (IAM) is a w-knot diagram invariant which can be used to recover the Alexander polynomial. However, not much is known about the IAM itself. In particular, it was not known if it is a knot invariant. In my talk, I will describe knots, their generalization to w-knots, the IAM, and my results on the question of whether it's a knot invariant.

Title: An Algebraic Definition of the Bridge Number of a Knot
Presenter: Paul Villanueva
Affiliation: Iowa State University
Abstract: We define the Wirtinger number of a knot. The Wirtinger number is a knot invariant equal to the minimum number of generators of the fundamental group of the knot complement over all knot diagrams where every relation is a Wirtinger relation. The bridge number of a knot is a classic knot invariant measuring the minimum number of local maxima across all equivalent representations of the knot. We prove that the Wirtinger number of a knot equals its bridge number. This result leads to a computational technique by which we have added the bridge number of approximately 50,000 prime knots of up to 12, 13, and 14 crossings to the knot table, as well as establishing tight upper bounds on the bridge number of prime knots of 15 and 16 crossings. Additionally, this result suggests a new approach to Cappell and Shaneson's Meridional Rank Conjecture.

Title: Differential Forms
Presenter: Khoi Vo
Affiliation: California State University, Long Beach
Abstract: In elementary calculus, when we studied $2, 3$ dimensional objects, we have seen in Green's Theorem and Stroke's Theorem the relationship between integration over a bounded region with the integration over the boundary alone. It is then of course reasonable to expect the same relationships occur in higher dimensions, or they can be generalized to other types of objects such as smooth manifolds.In this paper, we will introduce the necessary background to build up such generalizations. One of the difficulty in the process of generalization is the formal definition of the infinitesimal expression fdx or $(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y})dxdy$, which was usually "scrubbed under the rug" when first introduced in a calculus course. Here we need to build up rigorous definitions with terminology such as **forms, differential forms, chains**..etc.. After that, the corresponding Classical Theorems in Calculus, using these terminology, will be extremely elegant and beautiful. It is then where we can fully appreciate the purpose of all the new terminologies introduced earlier.

** Invited Faculty Speaker
* Distinguished Graduate Student

Poster Abstracts

Title: Coseparation with respect to an interior operator in topology
Presenter: Alexis Carrillo Blanquicett
Affiliation: University of Puerto Rico - Mayaguez
Abstract: Motivated by the results obtained by G. Castellini and E. Murcia in the paper "Interior Operators and Topological Separation", the notion of I-coseparation for an interior operator I in topology is introduced. A few examples that illustrate the behavior of this notion is presented, for concrete interior operators in topology. Subsequently, it is determined which topological properties are closed under this notion, hence it is obtained in particular that the I-coseparated topological spaces are closed under direct images of continuous functions and quotient spaces but they are not closed under topological sums and topological subspaces.

It is proved that the notion of I-coseparation generates a Galois connection between the class of all interior operators in topology and the conglomerate of all the subclasses of topological spaces. Using this result a commutative diagram of Galois connections that shows the relationship between the notions of I-separation and I-coseparation is presented. Finally it is proved that a characterization of the I-coseparated spaces in terms of separators, analogous to the presented in the paper studied, for the notion of I-separation, is not possible.

Title: Virtual Spatial Trivalent Graphs and Braids
Presenter: Abigayle Dirdak
Affiliation: California State University, Fresno
Abstract: A virtual spatial trivalent graph diagram (virtual STG diagram) is a trivalent graph immersed in a plane, which contains finitely many transverse double points, each of which has information of over/under or virtual crossings. We regard virtual STG diagrams as combinatorial objects up to an equivalence relation induced by certain combinatorial moves for virtual STG diagrams. Then a virtual spatial trivalent graph is the equivalence class of a virtual STG diagram. Moreover, we say that two virtual STG diagrams are equivalent if they belong to the same equivalence class.

A virtual trivalent braid is a braid similar to the notion of a classical braid, but may contain trivalent vertices and virtual crossings, in addition to classical crossings. The closure of a virtual trivalent braid with n endpoints on top and n endpoints in the bottom is a virtual STG diagram. Therefore, we can study virtual trivalent braids to gain information about virtual spatial trivalent graphs.

In this presentation we describe our method for converting any virtual STG diagram into an equivalent diagram in braid form. We also provide conditions for having two virtual trivalent braids whose closures yield equivalent virtual STG diagrams. In other words, we provide Alexander- and Markov-type theorems for virtual spatial trivalent graphs and virtual trivalent braids.

Title: An Euler phi function for the Eisenstein integers and some tantalizing applications
Presenter: Emily Gullerud
Affiliation: University of Wisconsin - Eau Claire
Abstract: We generalize the Euler phi function to the Eisenstein integer ring $\mathbb{Z}[\rho]$ where ρ is the primitive third root of unity $e^{2\pi i/3}$ by finding the order of the group of units in the ring $\mathbb{Z}[\rho]/(\theta)$ for any given Eisenstein integer θ. As one application, we prove that the celebrated Euler-Fermat theorem holds for the Eisenstein integers. As another application, we investigate a sufficiency criterion for when certain unit groups $\left(\mathbb{Z}[\rho]/(\gamma^n)\right)^\times$ are cyclic where γ is prime in $\mathbb{Z}[\rho]$ and $n \in \mathbb{N}$, thereby generalizing well-known results of similar applications in the integers and some lesser known results in the Gaussian integers.

Title: Computer Modeling and Waldo
Presenter: Chance Hamilton
Affiliation: Florida Gulf Coast University
Abstract: We implement existing machine learning and image processing techniques to develop an open source

computer program that is able to play Where's Waldo. Our program utilizes trained Support Vector Machines (SVMs) and low level color filters to create a decision matrix that predicts the likelihood that a given character is located inside each 48×48 pixel box. Our Python program has achieved a 100% success rate of finding Waldo and the Wizard with few false positives.

Title: The Shape of Data
Presenter: Karen Medlin
Affiliation: Borough of Manhattan Community College
Abstract: This poster illustrates findings from a topological investigation into a classic example of machine learning called the Perceptron (Rosenblatt, 1957). With the Perceptron proven to generate results in n-dimensional Euclidean space, are there additional shapes to be illuminated with this algorithm other than identifying whether or not the input data is linearly separable? Working with persistence, as developed by Carlsson, Ghrist, and their collaborators, we proposed a model relating the number of iterations of the Perceptron to persistent homology, and are investigating other topological connections to the Perceptron.

Title: Distributions of Matching Distances in Topological Data Analysis
Presenter: Taylor Okonek and Nikesh Yadav
Affiliation: St. Olaf College
Abstract: As topological data analysis continues to develop as a field of study and is used more frequently in scientific research, it will be important to understand whether or not certain features in data are significant as opposed to simply random noise. While progress has been made on statistical techniques for single-parameter persistence, the case of two-parameter persistence, which is highly desirable for real-world applications, has been less studied. The goal of this study was to determine a measure of statistical significance for point cloud data through the analysis of null-type distributions of matching distances computed from two-dimensional persistence modules. These null-type distributions can be extrapolated to the creation of null hypotheses for real-world data. This study used two-parameter persistent homology to analyze the structure of randomly produced point clouds with known geometric structure, such as two disjoint circles with varying levels of noise. Calculations were performed with the two-parameter persistence homology software RIVET and the Hera software for algebraic distance calculations. Results include observations of how differences in geometric structure of point clouds affect the matching distance between persistence modules. These results will be useful in topological data analysis in many important applications, including the structure of various polymers, plant root systems, and image analysis.

Title: Polygonal Wheel Graphs and Links
Presenter: Dawn Paukner
Affiliation: The University of Wisconsin-Eau Claire
Abstract: Our research aims to discover more relationships between graph theory and knot theory. We define and use what we call polygonal wheel graphs to create a unique class of alternating knots and links. We were able to discover a relationship between the polygons in our wheel graphs and different knot and link invariants. More specifically, our poster will discuss the connection between these graphs and the determinant, colorability, component number, genus, and Alexander polynomial of the links created.

Title: Characteristic polynomials and eigenvalues for a family of tridiagonal real symmetric matrices
Presenter: Rita Post
Affiliation: The University of Wisconsin-Eau Claire
Abstract: We explore a certain family $\{A_n\}_{n=1}^{\infty}$ of $n \times n$ tridiagonal real symmetric matrices. After deriving a three-term recurrence relation for the characteristic polynomials of this family, we find a closed form solution. The coefficients of these characteristic polynomials turn out to involve the diagonal entries of Pascal's triangle in an attractively inviting manner. Lastly, we explore a relation between the eigenvalues of various members of the family. More specifically, we give a sufficient condition for when $\texttt{spec}(A_m)$ is contained in $\texttt{spec}(A_n)$.

Title: Virtual spatial trivalent graphs and their braids
Presenter: Erica Sawyer
Affiliation: The Evergreen State College
Abstract: A virtual spatial trivalent graph diagram (virtual STG diagram) is a trivalent graph immersed in a plane, which contains finitely many transverse double points, each of which has information of over/under or virtual crossings. We regard virtual STG diagrams as combinatorial objects up to an equivalence relation induced by certain combinatorial moves for virtual STG diagrams. Then a virtual spatial trivalent graph is the equivalence class of a virtual STG diagram.

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In this presentation, we describe our method for converting any virtual STG diagram into an equivalent diagram in braid form. We also provide conditions for having two virtual trivalent braids whose closures yield equivalent virtual STG diagrams. In other words, we provide Alexander- and Markov-type theorems for virtual spatial trivalent graphs and virtual trivalent braids.

Title: Some results on the conjugacy classes in the hyperoctahedral group and a splitting criterion conjecture
Presenter: McKenzie Scanlan
Affiliation: The University of Wisconsin-Eau Claire
Abstract: We examine elements in both the hyperoctahedral group $\mathbb{G}(2,1,n)$ and an analogue of the alternating subgroup $A_n \subseteq S_n$ in $\mathbb{G}(2,1,n)$—namely, the orientation-preserving symmetries, which we denote $\mathbb{A}(2,1,n)$. In particular, we investigate their corresponding conjugacy classes and centralizers. We employ known symmetric group results on both the size of a conjugacy class and the size of a centralizer for a particular element $\sigma \in S_n$ to determine an analogous result in the hyperoctahedral group setting. We study the previously known splitting criterion for conjugacy classes in the alternating subgroup of the symmetric group with the ultimate goal of finding the analogue in the hyperoctahedral group and its alternating subgroup. We use Mathematica to begin this investigation.

Title: Finding Automorphisms in Hopf Algebra Dimension 12
Presenter: Abby Vorhaus
Affiliation: United States Military Academy
Abstract: A Hopf algebra H over the field k is an algebra and coalgebra over k with an antimorphism structure called the antipode. In this project, we will focus on a semi-simple Hopf algebra of dimension 12 over the field $\mathbb{C}$ which we denote H_{12}. We programmed both the algebra and coalgebra structures of H_{12} in Mathematica. Using our Mathematica code as well as techniques used by D. Sage and M. Vega for a Hopf algebra of dimension 8, we provide an explicit formulation for automorphisms in H_{12}.

Special thanks to the following USTARS sponsors, partners, and supporters:

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